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Essays on Contract Design: Delegation and Agency Problems, and Monitoring under Collusion

A dissertation submitted in partial satisfaction of the requirements for the degree Doctor of Philosophy in Economics

by

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This dissertation is dedicated to

my wife Carola and

my daughter Florencia.

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ABSTRACT OF THE DISSERTATION

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by

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The first chapter analyzes, in the context of one-sided delegated bargaining, how a principal (a seller) should design the delegation contract in order to provide proper incentives for her delegate (an intermediary) *AND* gain strategic advantage against a third party (a buyer). In a context with risk-neutral players, moral hazard and adverse selection problems in the delegation relationship, and absence of commitment effect, a linear contract is optimal. When delegation contracts have commitment value, the seller can gain substantially by imposing a minimum price, above which she pays the delegate a commission. The interaction between commitment (through minimum price) and incentives depends on the nature of the agency problem. Incentives and commitment are substitute when the delegate's unobservable effort improves

his bargaining position, but are neither substitute nor complement when his effort increases chances of finding a buyer. In most cases, the seller's strategic manipulation of the delegation contract may cause bargaining failures between the delegate and the buyer.

The second and third chapters examine, in a principal-monitor-agent framework, the use of the timing to monitor the agent as a contracting instrument. First, we show that, under some conditions, the optimal contract with an ex post monitor depends on obtaining a signal after the agent exerted effort and not on what is being monitored (agent's effort or productivity). This result extends to a self-interested monitor for any possible level of collusion between the agent and the monitor (i.e., information concealment or manipulation). In the third chapter we analyze the optimal monitoring timing in different collusion environments. Ex ante monitoring (supervising) is optimal for weak punishment and an informative signal, and ex post monitoring (auditing) is optimal otherwise. This result extends to collusion with hard and non-forgeable information, but may be reverted when information is hard and forgeable or soft. In these cases, collusion imposes costs to both auditing and supervising and, under some circumstances, supervising is more likely to be optimal when the signal is noisy even for unbounded punishments, or when punishment schemes are weak (independently of the signal's noise).

Chapter 1

Delegated Bargaining: Agency Problems and Commitment

1.1 Introduction

In many economic situations, delegates are hired to play games on behalf of their principals. Consider the owner of a car dealership, who hires sales managers to sell cars to customers. Between the owner and her managers, there often exist various types of agency problems (moral hazard, adverse selection and combinations of both), with which the principal-agent literature has extensively dealt. However, in most of this literature, the game the agent is hired to play with other parties (e.g., bargaining between sales managers and car buyers) is suppressed in the studies of optimal agency contracts (the agent' actions alone determine the principal's payoff subject to perhaps exogenous randomization by nature). On the other hand, since Schelling [52], it has long been recognized that the principal may gain strategic advantages against a third party by properly designing a contract for the agent. In the context of car dealership, the agency contract between the dealership owner and her sales manager may affect how the sales manager and car buyers bargain and hence ultimately the terms of trade. A large amount of subsequent work has investigated when this commitment effect can arise and its implications in various economic situations.[1] But not much attention has been paid to the interactions between agency problems and commitment considerations in the delegation relationship. In this chapter, we analyze such interactions in an important class of delegation games, delegated bargaining.

Specifically, we study the following one-sided delegation game. A seller of one indivisible good hires a delegate (an intermediary) to sell the good for her. They sign a contract, which becomes public knowledge. At the time the agency contract is signed, neither the seller nor the delegate knows the valuation of the (potential) buyer but they know its distribution. After exerting some unobservable "sales efforts", the delegate meets a buyer and then finds out the buyer's valuation. Then they bargain over a price, so bargaining is conducted under complete information. If the delegate and the buyer agree on a price, the buyer gets the good and makes the payment, and the delegate delivers the payment to the seller. The seller pays the delegate a wage according to the delegation contract. The seller only observes the sale revenue the delegate brings back to her. We assume that the delegate and the buyer cannot collude and the delegate cannot hide money from the seller. All the players are assumed to be risk-neutral. The model is obviously stylized, but seems to capture

[1] See, e.g., Vickers [59], Fershtman and Judd [18, 17] , Sklivas [54], Dewatripont [13], Gal-Or [21, 22, 23], Fershtman, Judd and Kalai [19], Katz [32], Hermalin [28, 29], Caillaud, Jullien and Picard [8], Martimort [45], Baye, Crocker and Ju [4], Laffont and Martimort [40], Fershtman and Kalai [20], Corts and Neher [10], Kockesen and Ok [37].

some of the essential features in the car dealership example and many other similar situations with trade intermediaries.

We suppose that there are both moral hazard and adverse selection problems in the delegation relationship. That is, the delegate's effort is not observable to the seller; and furthermore, the delegates can differ in their disutility of effort, which is not observable to the seller either. Ignoring the commitment effect of delegation contracts, we can characterize the seller's optimal mechanism. Using the insights of the earlier literature (e.g., Holmstrom and Milgrom [30], Laffont and Tirole [44], and McAfee and McMillan [46]), we show that a contract linear in sales revenue can implement the seller's optimal mechanism under certain mild conditions. This is done in Section 1.2. The case without commitment considerations serves as a useful benchmark.

We then turn to the case with commitment effect. We assume that delegation contracts are perfectly observable to potential buyers and cannot be renegotiated.[2] If the seller knew exactly the buyer's valuation, then she could achieve "full commitment" by using a "target contract". A target contract requires the delegate to

[2] Several papers, e.g., Katz [32], Caillaud, Jullien and Picard [8], Dewatripont [13], Fershtman and Kalai [20], Corts and Neher [10], Kockesen and Ok [37], have examined whether delegation still has commitment power if delegation contracts are not perfectly observable or can be renegotiated secretly. By and large, these papers show that unobservability and renegotiation of delegation contracts *limit but do not eliminate* the commitment effects of delegation. Interestingly, asymmetric information between the principals and their delegates is usually necessary for delegation contracts to have commitment effects when renegotiation is allowed. In practice, the seller can maintain the credibility not to renegotiate delegation contracts for reputation reasons if she hires the delegate to do repeated sales with different customers or hires multiple delegates to conduct similar sales. See more discussions in Section 1.4.

get a certain price for the good, otherwise he is paid nothing or even faces some penalty. Without uncertainty, the seller can set the price target exactly equal to the buyer's valuation, which commits the delegate to get this price and leave the buyer with no surplus.[3] In reality, the seller often does not observe directly the buyer's valuation, and the agency problems make it difficult for the agent to communicate his knowledge about the buyer perfectly to the seller. In such cases the target contracts are not feasible anymore, thus the commitment power of delegation contracts is limited and the seller usually cannot achieve full commitment. In Section 1.3, we show that the seller can still achieve a substantial amount of commitment power by imposing a minimum price with a linear sharing contract. A minimum price can give the delegate bargaining advantage because it raises his threat point. With a standard Rubinstein bargaining model, it can be easily shown that the higher the minimum price, the higher the final sales price, *provided that the buyer's valuation is higher than the minimum price*. We derive the seller's optimal minimum price and optimal commission rate jointly. Under fairly general conditions, the seller sets a minimum price that is strictly greater than the lower bound of the buyer's valuations. This means that when the buyer's valuation is below this minimum price, the delegate and the buyer cannot reach a deal despite that there are positive gains from trade. We then investigate how the minimum price interacts with the commission rate to

[3] Fershtman *et al.* [19] show that with target contracts, any Pareto optimal outcome in a principals-only game can be achieved when (1) every principal can hire a delegate; (2) contracts are observable and not renegotiable; and (3) there are no agency problems. Kahenmann [31] reaches similar conclusions in the context of Rubinstein bargaining.

provide an optimal balance between commitment and incentive considerations.

One excuse of our focus on contracts linear in sales revenue above minimum price is simply that analysis of more complicated schemes is not tractable. This focus is also motivated by the observation that it is commonly used in real life. In car dealerships, "most salesmen are paid a commission which is usually 25-30 % of the gross profit (based on dealer invoice, not incentives) on every car they sell." (Eskeldson [14], p. 46) To us, the most interesting part of the agency contracts for car salesmen is what commission is based on. As shown in Section 1.2, when agency problems are the only consideration, the optimal contract requires commissions to be based on the seller's profit, i.e., sales revenue minus the seller's cost. However, in the U.S. car dealership business, the real cost of a car to the dealer is usually not its invoice price. Car manufacturers typically offer dealers "holdbacks", which pays back the dealers a certain percentage (usually 2-3 %) of the invoice prices when cars are actually sold. In addition to dealer holdbacks, car manufacturers offer many other different kinds of incentives (dealer rebates, volume discounts, credit discounts, etc.) from time to time (and can vary across dealerships). With all these provisions from car manufacturers taken into account, the real cost per car for the dealer is substantially lower than its invoice cost (of course, a small operational cost per car has to be added). Since commissions of car salesmen are calculated on the basis of invoice prices but not on dealer's real cost, car salesmen will not be willing to sell cars under invoice prices. Thus invoice prices in the car dealership example can be viewed as the minimum prices in our model, and our analysis provides a justification for using invoice prices to calculate commissions for car salesmen.

We find that the nature of the agency problem affects how the seller should optimally balance commitment and incentives. Specifically, we consider two kinds of moral hazard problems by the delegate. In the first scenario, the delegate exerts “bargaining effort” which increases his bargaining power against the buyer (e.g., doing research about the customers and the product, taking courses to improve bargaining skills). In this case, commitment through minimum prices and incentives for the delegate are *substitutes* for the seller, that is, higher minimum prices are associated with lower incentives for the delegate and hence lower effort by the delegate. As a result, high type agents are given more discretion in making deals with customers and are held responsible for the outcomes to a greater degree. In another scenario, the delegate exerts “marketing effort” which increases the chance that he finds a buyer (e.g., doing advertisement, providing good services, having clean showrooms). With “marketing effort”, commitment through minimum prices and incentives for the delegate are neither substitutes nor complements. This means that for some exogenous changes in the environment, higher minimum prices are associated with higher incentives for the delegate and hence higher effort by the delegate; but for some other exogenous changes in the environment, minimum prices and incentives move in the opposite directions.

We also find that strategic delegation may lead to bargaining failures under general conditions in our model.[4] In our model, the delegate and the buyer bargain under complete information, yet sometimes they fail to reach agreements because the delegate is pre-committed by the seller to bargain aggressively all the time. This is closely related to Haller and Holden [27], who show that a heterogeneous group of people sometimes want to impose a super-majority ratification rule on the bargaining outcomes their delegate reached with a third party in order to gain strategic advantage. As a result, an agreement beneficial from the perspective of the median voter may fail to be reached. The main difference between Haller and Holden [27] and this work is that while they focus on intra-group heterogeneity among the principals, we focus on the agency problems in the delegation relationship. Crawford [11] formulated the idea that commitment by bargainers in the presence of uncertainty can lead to bargaining failures, but he abstracted away from the commitment instruments bargainers use.[5]

Hiring a delegate to bargain with the buyers is one of the trading mechanisms used in real life (but not examined in the literature). In Section 1.4, we compare this

[4] That strategic delegation causes distortions is not new. For example, in oligopolistic competition, Fershtman and Judd [17] show that strategic delegation leads to lower price, lower profit but greater social surplus if oligopolists compete in Cournot fashion but the opposite is true if they compete in Bertrand fashion (see also Baye, Crocker and Ju [4], Vickers [59]).

[5] Studying a variation of Crawford [11], Muthoo [48] shows that without uncertainty about costs of revoking commitments, the bargaining outcome will be efficient. In another related paper, Cai [7] shows that the agency problems in the delegation relationship can cause bargaining inefficiency. Specifically, in Cai's model, a delegate bargains with a third party under complete information but faces reelection after the bargaining outcome becomes known to his constituency (principals). In this case, delay in reaching agreements can be used by the delegate as a signal to his principals that he is of "good type". In contrast to Cai [7], the agency problems in the delegation relationship do not directly cause bargaining inefficiency in our model. Rather, bargaining failures are caused by the seller's strategic manipulation of the delegation contract that commits the delegate to bargain aggressively.

mechanism with another commonly studied mechanism: standard monopoly pricing, whereby the seller commits to a fixed price (i.e., posted-price selling). Also in this section we discuss some of the factors that affects whether and how the minimum price can be credibly used as a commitment device to give the delegate bargaining advantage.

To study in more detail how the optimal mechanism responds to exogenous changes in the environment, in Section 1.5 we derive comparative statics of the model for the case of uniform distributions and quadratic cost functions. We present and compare results for three cases: no commitment effect, commitment effect with bargaining effort, and commitment effect with marketing effort. For concreteness, Section 1.5 also gives some numerical examples where the model is explicitly computed. In one seemingly reasonable configuration of parameter values, there is a 39% probability that the delegate will not reach a deal with a buyer because of the seller's minimum price policy, resulting in about welfare loss of 16% of the total social surplus. In this case, the seller's expected payoff is more than 65% higher than that if she did not take advantage of the strategic value of delegation contract.

Finally, Section 1.6 offers some concluding remarks.

Fershtman and Judd [18] is the first model that studies how optimal contracts should respond to both agency problems and commitment considerations. Specifically, they consider a double-sided delegation game in which two managers are hired by their owners to compete with each other in an oligopolistic situation. In their model, like ours, delegation contracts are public information and not renegotiable. Unlike in our model, there is only moral hazard problem in the delegation relationship and the owners are more risk-averse than the managers are (so without commitment considerations, the owners should sell the firms to the managers). Fershtman and Judd show that to take advantage of the commitment power of the delegation contracts, the owners "over-compensate" the managers for success and thus bear more risk than efficient risk-sharing. In fact, the incentives for the managers are so strong that an owner is better off if her manager fails. Caillaud *et al.* [8] analyze a duopolistic competition model with double sided delegation in the presence of both agency problems and commitment. There is only adverse selection in their model (action is contractible) and delegation contracts are renegotiable. Caillaud *et al.* show that public but renegotiable delegation contracts still have commitment effects because they impose restrictions on the possible renegotiation outcomes in the presence of asymmetric information. The focus and analysis of their paper are quite different from ours.

1.2 The Basic Model Without Commitment Effect

The model consists of three *risk-neutral* parties: a seller (P), a delegate (D), and a buyer (B). The seller hires the delegate to sell a good to the buyer. The cost of the good to the seller is normalized to be zero. The delegate's reservation utility is U_0. At the time the seller contracts with the delegate, the valuation of the buyer for the good is unknown to both the seller and her delegate. Their common belief about the valuation is given by a probability distribution $G(s)$ with an everywhere positive density function $g(s)$, where $s \in [\underline{s}, \bar{s}]$ $(0 \leq \underline{s} < \bar{s})$ is the buyer's valuation.

For tractability, we suppose that when the delegate meets the buyer, the delegate finds out the buyer's valuation. So they bargain over a price without any information problem. The exact bargaining game will be specified later. For now, let us just say that the delegate can get a share of r of the total surplus for the seller in the equilibrium of the bargaining game. We assume that before bargaining with the buyer, the delegate can exert efforts to increase revenue for the seller. We consider two kinds of effort. The first is "bargaining effort", which increases the delegate's share for any fixed surplus. In this case we write the delegate's share r as a function of his effort e; and we assume that for all e, $r(e) \in (0,1)$, $r'(e) > 0$ and $r''(e) \leq 0$. Another type of effort is "marketing effort", which increases the probability that the delegate finds a buyer. Conditional on finding a buyer, the delegate will get a fixed share of r_0. We write the probability of finding a buyer p as a function of the delegate's marketing effort; and assume $p(e) \in (0,1)$, $p'(e) > 0$ and $p''(e) \leq 0$. For a fixed surplus s, the expected price the delegate can get in the case of bargaining effort is $x = r(e)s$ while in the case of marketing effort is $x = r_0 p(e)s$, so there is no real difference in the expected price between these two types of efforts. Indeed, in this section we ignore the commitment effect of delegation contracts, the two cases are identical (and we will use the bargaining effort interpretation). But in the next section, when commitment effect is present, the two cases will yield somewhat different results.

The delegate incurs effort cost of $C(e,t)$, where t is his "type" that characterizes his disutility of effort. We make the following standard assumptions on $C(e,t)$: (i) $C(e,t)$ is strictly increasing and convex in e, $C_e = \partial C/\partial e > 0$ and $C_{ee} = \partial^2 C/\partial e^2 > 0$; and (ii) higher types have lower effort cost and lower marginal effort cost, that is, $C_t = \partial C/\partial t < 0$ and $C_{et} = \partial^2 C/\partial e \partial t < 0$. The seller does not observe either the effort or the type of the delegate. Therefore, there are *both moral hazard and adverse selection* in the delegation relationship. At the time the seller is contracting with the delegate, the seller knows that the delegate's type is drawn from a distribution function $F(t)$ with density function $f(t) > 0$ for every $t \in [\underline{t}, \overline{t}]$, the domain of t.

Throughout this chapter, we make the following standard assumption on $F(t)$:

Assumption 1 *The distribution of types $F(t)$ satisfies the monotone hazard rate property, that is, $f(t)/[1 - F(t)]$ is increasing in t.*

This assumption is satisfied by common distributions, such as uniform or log-normal.

For simplicity, we also make the following technical assumptions:

Assumption 2 *(i) C_{et} is a negative constant; (ii) $r'(e)$ and $p'(e)$ are positive constants.*

These two technical assumptions ensure that the agent's expected payoff function is concave. The results presented here will not be affected if alternatively we make more general but less intuitive assumptions involving C_{eet}, C_{ett}, r'' and p''. By Part (ii), we will write $r(e) = r_0 + r'e$ and $p(e) = p_0 + p'e$, where r' and p' are positive constants.

We assume that all the three players are risk-neutral. Suppose the total surplus is s, and the delegate obtains x (i.e., the price is x) for the seller, and the seller pays the delegate a wage of w. Then the seller's utility is $U_P = x - w$, the delegate gets a utility of $U_D = w - C(e,t)$, and the buyer's utility is $U_B = s - x$.

The timing of the game is as follows. At date 0, the seller (she, henceforth) hires a delegate (he, henceforth), whose type is unknown to her. She offers a menu contract to him, which is observable and non-renegotiable. At date 1, the delegate decides whether to continue the game or quit. If he stays in the game, then at date 2, he chooses an effort level e. At date 3, the delegate meets the buyer, learns the buyer's valuation of the good, and they bargain over a price. Finally, once a deal is reached, the delegate gives the sale revenue to the seller, who then pays the delegate according to their contract. Throughout the game, the seller can only observe the sale revenue. This implicitly assumes that the delegate and the buyer cannot collude, otherwise it would be easy for the buyer to hide some of the revenue. While collusion between the delegate and the buyer is a real concern for the seller and is an interesting issue to analyze, it is beyond the scope of this work and thus assumed away. In some cases, reputation concerns of the delegate or legal constraints may help control collusive behavior of the delegate.

Now we turn to the specification of the bargaining game. The main results hold for the standard alternating-offer bargaining games such as Rubinstein [51] or Binmore, Rubinstein and Wolinsky [5] and equivalently the cooperative solution concept Nash Bargaining Solution.[6] For concreteness, we adopt the bargaining game of Binmore, *et al.* [5]: they alternate in making offers and there is a small exogenous probability ρ_d (respectively, ρ_b) that bargaining will break down whenever the delegate (respectively, the buyer) rejects an offer. When bargaining breaks down, the total surplus disappears (e.g., out of his own control, one bargainer walks out of the room and never returns). With this bargaining game, the effect of the delegate's bargaining effort is to reduce ρ_d (that is, having better control over the bargaining process). Bargaining could last infinite rounds if there is no agreement or breakdown. Suppose the delegate moves first by making an offer to the buyer.

When the delegate bargains with the buyer on behalf of the seller, how much the delegate will get in equilibrium can be affected by the contract between the seller and the delegate. Our main focus is precisely on how this commitment consideration affects the design of the delegation contract. To make meaningful comparisons, in this section, we first analyze the optimal contract design problem while ignoring the commitment effect. So for now, we suppose that for some reason the buyer bargains with the delegate as if the delegate were representing himself. This could happen when the buyer does not know whether the delegate is representing himself or acting

[6] See Osborne and Rubinstein [50] for discussions about the link between non-cooperative alternating-offer bargaining games and the Nash Bargaining Solution.

as the agent for the seller.[7]

After meeting a buyer, suppose the delegate finds out that the buyer's valuation is s. The following lemma is a standard result from Binmore *et al.* [5]:

Lemma 1 *When the delegate represents himself, the bargaining outcome in the unique subgame perfect equilibrium is such that the delegate and the buyer reach an agreement without delay and the sales price is $x = rs$, where $r = \rho_b/[1-(1-\rho_b)(1-\rho_d)]$.*

Since the equilibrium share of the delegate r decreases in ρ_d, it increases in his bargaining effort e, which was assumed before. Alternatively, we could use the standard Rubinstein bargaining model, which would have $r = (1-\delta_b)/(1-\delta_d\delta_b)$, where δ_d and δ_b are the discount factors of the delegate and the buyer respectively. Then the effect of delegate's bargaining effort would be to increase δ_d (i.e., improving patience). Or, we could use the Nash Bargaining Solution and suppose the seller's relative bargaining power is r while the buyer's is $1-r$, then maximizing $rln(x)+(1-r)ln(s-x)$ gives $x = rs$.

For future comparisons, let us consider first the case in which both the delegate's effort and type are observable to the seller. For a delegate of type t, the seller asks him to exert effort $e(t)$ and pays him a wage that covers his effort cost and his reservation utility. So $w(t) = C(e(t),t) + U_0$. Then the seller's expected profit is simply $EU_P = \int_{\underline{s}}^{\bar{s}}[r(e)s - w(t)]dG(s) = r(e)E(s) - C(e,t) - U_0$, where $E(s) = \int_{\underline{s}}^{\bar{s}} sdG(s)$. So the

[7] Fershtman and Kalai [20] show that when the third party (here the buyer) either does not know whether or not the delegate is representing himself or simply does not observe the details of the contract, no commitment effect is still a trembling hand sequential equilibrium.

optimal effort $e_{FB}(t)$ for the seller satisfies the following condition:

$$r'E(s) = C_e(e_{FB}(t), t) \tag{1.1}$$

where subscripts are partial derivatives with respect to the corresponding variable (that is, $C_e = \partial C/\partial e$). By our assumptions, the second-order condition is satisfied and the solution to Equation (1.1) is unique. Also the optimal effort $e_{FB}(t)$ increases in the delegate's type t. Note that since both the delegate's effort and type are observable, there is no need to make wage contingent on sale revenue.

When the seller does not observe the delegate's effort and type, the optimal contract design problem can be analyzed in the mechanism design framework. By the revelation principle, it is without loss of generality to focus on direct revelation mechanisms in which the delegate is provided proper incentives to reveal his type truthfully and behave obediently. In a direct revelation mechanism, a seller's mechanism consists of a wage schedule $w(\hat{t}, x)$ that depends on the delegate's announced type $\hat{t}$ and the sale revenue x he eventually brings back, and a recommendation of effort level $e(\hat{t})$ that depends only on his announced type $\hat{t}$. Given the seller's mechanism, the delegate of type t chooses an announcement of type $\hat{t}$ and an effort level to maximize his expected utility $U_D = \int_{\underline{s}}^{\bar{s}} w(\hat{t}, x)dG(s) - C(e, t)$.

Formally, the seller's problem is to find a wage schedule $w(\hat{t}, x)$ and a recommendation $e(\hat{t})$ that solves

$$\max_{\{w(t,x),e(t)\}} EU_P = \int_{\underline{t}}^{\bar{t}} \int_{\underline{s}}^{\bar{s}} [x - w(t, x)]dG(s)dF(t) \tag{1.2}$$

subject to

(i) $(t, e(t)) \in argmax_{\{\hat{t},e\}}\ U_D = \int_{\underline{s}}^{\bar{s}} w(\hat{t}, x) dG(s) - C(e, t)$

(ii) $U_D(t) = \int_{\underline{s}}^{\bar{s}} w(t, x) dG(s) - C(e(t), t) \geq U_0$, $\forall t$

(iii) $x = r(e(t))s$, $\forall s$

Condition (i) is the incentive compatible constraint for the delegate. It states that he finds it optimal to report his true type and to choose the recommended level of effort. The interim participation constraint (condition (ii)) requires that the optimal contract has to ensure the delegate at least his reservation utility. Finally, condition (iii) describes the bargaining outcome for every possible buyer's valuation when the commitment effect of delegation contract is ignored.

The mechanism design problem can be solved in two steps. In the first step, we characterize the conditions for an optimal mechanism; and then in the second step we find contracts that implement the optimal mechanism. The results of this section and their derivation closely follow McAfee and McMillan [46] (see also Laffont and Tirole [44]).

To characterize the conditions for an optimal mechanism, suppose the seller can observe the delegate's effort but not his type and therefore can force upon him an effort schedule $e(\hat{t})$. Then the IC condition (i) is reduced to truth-telling only. Using the Envelope Theorem and integration by parts, one can simplify the mechanism design problem to (technical details in the Appendix):

$$\max_{\{e(t)\}} \int_{\underline{t}}^{\bar{t}} \left\{ r(e)E(s) - C(e,t) + C_t(e,t)[\frac{1-F(t)}{f(t)}] \right\} dF(t) - U_0 \tag{1.3}$$

Let $e^*(t)$ be a solution to Equation (1.3). Then it has to satisfy the following first-order condition:

$$r'E(s) = C_e(e^*, t) - C_{et}[\frac{1 - F(t)}{f(t)}] \tag{1.4}$$

The following proposition gives the (sufficient) conditions for an optimal mechanism.

Proposition 1 *If a wage contract $w(\hat{t}, x)$ can induce the delegate to (i) truthfully reveal his type, and (ii) choose $e^*(t)$, and guarantees him the reservation utility, then the mechanism $\{w(\hat{t}, x), e^*(t))\}$ is optimal.*

Proof: See the Appendix.

Comparing Equations (1.1) and (1.4), one can see that the optimal effort in the presence of agency problems $e^*(t)$ is lower than that under complete and perfect information ($e_{FB}(t)$) for all types but $\bar{t}$. This is because the term $C_{et}[1 - F(t)]/f(t)$ in Equation (1.4) is negative for all $t < \bar{t}$. This term is the information rent to the delegate. Because of asymmetric information between the seller and the delegate, the economic cost of effort to the seller consists of the direct effort cost to the delegate $C(e, t)$ and the information rent. Equation (1.4) then simply says that marginal benefit of effort equals marginal cost of effort. Since the information rent increases the marginal cost of effort, the optimal level of effort should be lower.

The next step is to find contracts that satisfy all the conditions in Proposition 1. Consider the following contract that is linear in sale revenue. If the delegate makes a sale, his wage is given by

$$w(\hat{t}, x) = \alpha^*(\hat{t}) + \beta^*(\hat{t})x \tag{1.5}$$

where $\alpha^*(\hat{t})$ and $\beta^*(\hat{t})$ are

$$\alpha^*(\hat{t}) = C(e^*(\hat{t}), \hat{t}) - \int_{\underline{t}}^{\hat{t}} C_t(e^*(\nu), \nu)d\nu - \frac{C_e(e^*(\hat{t}), \hat{t})}{r'} r(e^*(\hat{t})) + U_0$$
$$\beta^*(\hat{t}) = \frac{C_e(e^*(\hat{t}), \hat{t})}{r'E(s)};$$

and if the delegate fails to sell the good, his wage is simply $w = \alpha^*(\hat{t})$. In other words, $\alpha^*(\hat{t})$ is an up-front payment made to the delegate when signing the contract but before conducting the sale.

First we need to do a consistency check: such a linear contract should yield the bargaining outcome as assumed in the mechanism design problem in Equation (1.2).

Lemma 2 *With any linear contract of the form $w(\hat{t}, x) = \alpha(\hat{t}) + \beta(\hat{t})x$ where $\alpha(\hat{t})$ is an up-front payment, the bargaining game has a unique subgame perfect equilibrium with the bargaining outcome being that the delegate and the buyer reach an agreement without delay and the sales price is $x = rs$, where $r = \rho_b/[1 - (1 - \rho_b)(1 - \rho_d)]$.*

Proof: By the standard result in the literature (Rubinstein [51], Binmore *et al.* [5], Osborne and Rubinstein [50]), the bargaining game has a unique subgame perfect equilibrium in which the two bargainers will reach an agreement without delay. Let x be the equilibrium price when the delegate makes an offer and y be that when the buyer makes an offer. Then the standard argument in the literature implies that

$$s - x = (1 - \rho_b)(s - y)$$

$$\alpha + \beta y = (1 - \rho_d)(\alpha + \beta x) + \rho_d \alpha$$

Solving these two equations gives the result of the Lemma. *Q.E.D.*

Lemma 2 confirms that any linear contract (and hence the contract in Equation (1.5)) is consistent with the no-commitment effect assumption made in this Section. The idea is simple. The up-front payment $\alpha(t)$ does not have any impact on the bargaining process since it is sunk before the bargaining game. What matters for the bargaining game is that the delegate gets $w = \beta x$ if the agreed price is x. But the bargaining outcome with this contract is the same as when the delegate is representing himself (in which case his utility is simply x), because a change of scale in the delegate's utility does not affect his behavior. Therefore, the bargaining outcome is $x = r(e)s$, $\forall s$.

The next proposition states that this linear contract actually implements the optimal mechanism.

Proposition 2 *The linear contract presented in Equation (1.5) implements the optimal recommended effort $e^*(t)$ and induces truthful report of type.*

Proof: See Appendix.

The intuition for the optimality of the linear contract is as follows. The seller needs to provide incentives to the delegate for him to tell the truth and follow the recommended effort. Because of risk-neutrality, these two tasks can be separately accomplished by the linear contract: The slope of the linear contract in Equation

(1.5) provides proper effort incentives while the constant takes care of truth-telling about type.

A simple corollary can be derived from Proposition 2:

Corollary 1 *In the optimal linear contract, the optimal effort $e^*(t)$ and the sharing term $\beta^*(t)$ are non-decreasing in type, and the constant term $\alpha^*(t)$ is non-increasing in type.*

Proof: See Appendix.

This corollary says that with the optimal linear contract, a more able delegate (who dislikes effort less) is provided stronger incentives and hence works harder than a less able one. In particular, it can be checked that the highest type delegate gets all the residual sale revenue ($\beta^*(\bar{t}) = 1$) and exerts the efficient effort ($e^*(\bar{t}) = e_{FB}(\bar{t})$). Since a more able delegate is rewarded a higher proportion of the sale proceeds, the fixed portion of his wage is smaller than that of a less able delegate. In fact, for delegates of sufficiently high types, their fixed portion is negative. The interpretation is that lower types opt for higher fixed wage and smaller commissions, while higher types choose higher commissions and pay fees to get the job (such as franchise fees). Since we assume away any commitment effect by the delegation contract in this section, it does not make a difference whether the fixed portion of the wage contract α is paid before or after the bargaining game. But for the purpose of comparison with later sections, we suppose α is paid up front when the delegate takes the job (accept the contract) but before bargaining with the buyer.

1.3 Commitment Effect Through Minimum Price

In the preceding section, we demonstrate that a linear delegation contract can implement the optimal mechanism for the seller IF delegation contracts have no commitment effect. But as the delegation literature has demonstrated, in general what kind of contracts the seller has for the delegate can affect the bargaining process between the delegate and the buyer. Hence in designing the delegation contract, the seller should take advantage of the contract's potential strategic value. In this section, we study how this commitment effect influences the seller's contract choice and explore its implications. Due to the complexity of the problem, we focus on contracts that still keep some linear structures but impose minimum prices on the delegate. Minimum prices seem to be commonly observed in practice (e.g., in car dealerships), and our analysis attempts to shed light on their optimal uses in connection with optimal delegation contracts. We analyze the case with "bargaining effort" and then the "marketing effort" case.

1.3.1 Bargaining Effort

The seller can do better by modifying the linear contract given in Equation (1.5) to take advantage of the commitment effect. Consider the following contract. If the delegate makes a deal, his wage is

$$w(\hat{t}, x) = \alpha(\hat{t}) + \beta(\hat{t})(x - z(\hat{t})); \tag{1.6}$$

and if he does not sell the good, his wage is $\alpha(\hat{t})$ (i.e., an up-front payment). Here $z(\hat{t})$

is a minimum price that the seller wants the delegate to obtain. According to this contract, the seller pays the delegate $\alpha(\hat{t})$ once they sign the contract. The delegate then goes to bargain with a buyer over a price. If the delegate brings back more than $z(\hat{t})$, then the seller pays him a commission β of what the delegate obtains in excess of the minimum price $z(\hat{t})$. Otherwise, if the delegate brings back less than $z(\hat{t})$, then he has to pay back money to the seller in the amount of $\beta(\hat{t})(z(\hat{t}) - x)$.[8] If the delegate does not strike a deal with the buyer, he is paid zero (he still keeps $\alpha(\hat{t})$ since it is paid before the negotiation). Remember we suppose that the seller can observe whether there is a deal and the terms of the deal if it is made. So, for example, the delegate is penalized if he sells the good for nothing ($x = 0$) but pays no penalty if he reaches no deal. Note that the contract in Equation (1.5) is a special case of the above contract with $z = 0$ for all $\hat{t}$.

If the delegate cannot get a price above the minimum price, then he will refuse to make a deal and obtain a payoff of $\alpha(\hat{t})$. So the delegate's choice not to make a deal effectively changes the linear contract of Equation (1.6) into a convex contract. It is this convexity that gives bargaining advantage to the delegate. Studying more complex convex contracts is difficult because the bargaining outcomes depend on the specific shapes of the delegate's payoff function.[9]

[8] Any amount of penalty for a sale price below the minimum price will have the same effect. See Lemma 3.

[9] Haller and Holden [27] present a simple example showing that convexifying the delegate's payoff function is always beneficial. They also offer several explanations why such contracts are not realistic. Long ago Sobel [55] pointed out that if bargainers were allowed to choose their payoff functions among concave functions, choosing linear functions would be a dominant strategy.

Assuming the contract is credible to the buyer, then it will affect the bargaining between the delegate and the buyer. The bargaining outcome under this contract is reported in the following lemma.

Lemma 3 *Suppose the delegation contract is given by Equation (1.6). Then the equilibrium outcome from the bargaining stage is* $x = r(e)(s - z(\hat{t})) + z(\hat{t})$*,* $\forall s \geq z(\hat{t})$*. When* $s < z(\hat{t})$*, there will be no agreement.*

Proof: When $s < z(\hat{t})$, there is no way the delegate can get a positive wage from a deal at the same time the buyer is not worse off, so there will be no agreement in this case. Suppose $s \geq z(\hat{t})$. Again, standard arguments of bargaining theory imply that there is a unique subgame perfect equilibrium. Let x be the equilibrium price when the delegate makes an offer and y be that when the buyer makes an offer. Then

$$\begin{aligned} s - x &= (1 - \rho_b)(s - y) \\ \alpha + \beta(y - z) &= (1 - \rho_d)[\alpha + \beta(x - z)] + \rho_d \alpha \end{aligned}$$

Solving these equations yields $x = r(s - z(\hat{t})) + z(\hat{t})$, where $r = \rho_b / [1 - (1 - \rho_b)(1 - \rho_d)]$. *Q.E.D.*

From Lemma 3, we can see that when $s \geq z(\hat{t})$, the seller gains an additional amount of surplus $(1 - r(e))z(\hat{t})$ purely from the commitment effect. And this commitment value is larger when the minimum price z is set higher, as long as it is not too high to prevent a deal. The idea is simple. Define $\tilde{s} = s - z(\hat{t})$. The delegate has to get at least $z(\hat{t})$ for the seller in order to get paid. So the "real" surplus he and the buyer can bargain over is $\tilde{s}$, of which the delegate should get $r(e)\tilde{s}$ given their relative bargaining power. Lemma 3 also points out the potential cost of using a minimum price as a commitment device. That is, the seller may go over the board and set a too high price target that prevents the delegate from reaching a deal with the buyer.

If the seller sets a minimum price $z \in [0, \underline{s}]$, then for any possible s the delegate and the buyer will reach a deal. Since commitment comes without cost for $z \in [0, \underline{s}]$, it seems that the seller should seek the maximum amount of commitment in this range. This intuition is verified in the following lemma.

Lemma 4 *For any $z < \underline{s}$, the seller can get a greater expected payoff by increasing the minimum price z. Therefore, the seller should set the minimum price not less than $\underline{s}$ for every $\hat{t}$.*

Proof: See Appendix.

Since the contract analyzed in the previous section corresponds to $z = 0$, Lemma 4 implies that the contract is not optimal for the seller when delegation contracts have commitment power.

Now the central question is whether the seller wants to set a minimum price higher than $\underline{s}$. The seller's mechanism design problem can be stated as

$$\max_{\{\alpha(t),\beta(t),e(t),z(t)\}} EU_P = \int_{\underline{t}}^{\bar{t}} \left\{ \int_{z(t)}^{\bar{s}} \left[x - \beta(t)(x - z(t))\right] dG(s) - \alpha(t) \right\} dF(t) \qquad (1.7)$$

subject to

(i) $(t, e(t)) \in argmax_{\{\hat{t},e\}}\ U_D = \alpha(\hat{t}) + \beta(\hat{t}) \int_{z(\hat{t})}^{\bar{s}} (x - z(\hat{t}))dG(s) - C(e,t)$

(ii) $U_D(t) \geq U_0$, $\forall t$

(iii) $x = r(e(t))(s - z(t)) + z(t)$, for $s \geq z(t)$, and 0 otherwise

(iv) $z(t) \in [\underline{s}, \bar{s}]$ for all t

As before, this problem can be solved in two steps. First we find the conditions for the optimal effort $e^B(t)$ and minimum price $z^B(t)$, (where the superscript B stands for "bargaining effort"). Following similar technical steps as in the proof of Proposition 1, we can rewrite the problem as:

$$\max_{\{e(t),z(t)\}} \int_{\underline{t}}^{\bar{t}} \left\{ r(e)E[s - z|s \geq z] + z[1 - G(z)] - C(e,t) + C_t(e,t)[\frac{1 - F(t)}{f(t)}] \right\} dF(t) - U_0 \tag{1.8}$$

where the argument (t) is suppressed in e and z, $E[s - z|s \geq z] = \int_z^{\bar{s}} [s - z] dG(s)$ and $z \in [\underline{s}, \bar{s}]$.

By point-wise differentiation of Equation (1.8), and assuming interior solutions (i.e., $z^B \in (\underline{s}, \bar{s})$), $e^B(t)$ and $z^B(t)$ must satisfy the following first-order conditions:

$$r' E[s - z^B|s \geq z^B] = C_e(e^B, t) - [\frac{1 - F(t)}{f(t)}]C_{et} \tag{1.9}$$

$$(1 - r(e^B))(1 - G(z^B)) - z^B g(z^B) = 0 \tag{1.10}$$

From Equation (1.10), one can see that z^B must be less than $\bar{s}$, since $\partial EU_P/\partial z = -\bar{s}g(\bar{s}) < 0$ at $z = \bar{s}$.

To implement the optimal mechanism, the next step is to find the optimal α and β that induce the delegate to report his true type and then choose the desired level of effort e^B. Let $\alpha^B(\hat{t})$ and $\beta^B(\hat{t})$ in contract (1.6) be such that:

$$\begin{aligned}\alpha^B(\hat{t}) &= C(e^B(\hat{t}),\hat{t}) - \textstyle\int_{\underline{t}}^{\hat{t}} C_t(e^B(\nu),\nu)d\nu - \frac{C_e(e^B(\hat{t}),\hat{t})}{r'}r(e^B(\hat{t})) + U_0 \\ \beta^B(\hat{t}) &= \frac{C_e(e^B(\hat{t}),\hat{t})}{r'E[s - z^B(\hat{t})|s \geq z^B(\hat{t})]}\end{aligned} \tag{1.11}$$

The next proposition says that the contract (1.11) implements the optimal level of effort $e^B(t)$.

Proposition 3 *The linear contract (1.11) with the optimal minimum price $z^B(t)$ implements the recommended effort $e^B(t)$ and induces the delegate to report his true type.*

If the seller's optimal minimum price turns out to be $\underline{s}$, then the first-order condition for the optimal effort, Equation (1.9), is reduced to

$$r'[E(s) - \underline{s}] = C_e(e^B, t) - [\frac{1 - F(t)}{f(t)}]C_{et} \tag{1.12}$$

Denote this solution by $\tilde{e}(t)$.

Proposition 4 *Suppose for some $\tilde{t} \in (\underline{t}, \bar{t}]$, $1 - r(\tilde{e}(\tilde{t})) > \underline{s}g(\underline{s})$. Then the seller will set the optimal minimum price $z^B(t)$ above $\underline{s}$ for any delegate of type in $[\underline{t}, \tilde{t}]$. As a result, delegates of these types fail to reach agreement with the buyer with positive probability. In particular, if $\tilde{t} \geq \bar{t}$, then all delegates face a positive probability of bargaining failure.*

Proof: First note that $\tilde{e}(t)$ is non-decreasing in t. Since r is increasing in e, $1 - r(\tilde{e}(t)) > \underline{s}g(\underline{s})$ for any $t \in [\underline{t}, \tilde{t}]$. Suppose that the seller chooses $z^B = \underline{s}$ and $\tilde{e}(t)$ as in Equation (1.12) for some $t \in [\underline{t}, \tilde{t}]$. From the first-order condition (1.10), the seller can increase her expected payoff by choosing a minimum price $z > \underline{s}$. Contradiction. *Q.E.D.*

Proposition 4 points out that the seller's strategic use of delegation contracts may result in bargaining failures. Note that the condition in Proposition 4 is sufficient but not necessary. To understand this condition, let us suppose that the buyer's valuation s is uniformly distributed on $[\underline{s}, \bar{s}]$. If $\underline{s} = 0$ or $\bar{s}$ is very large and $1 - r$ is bounded from below, then for any $t \in [\underline{t}, \bar{t}]$, the seller sets a minimum price above $\underline{s}$. Otherwise, let $r(\tilde{e}(\tilde{t})) = k$ and the condition in Proposition 4 is equivalent to $(1.5 - k)\Delta s > E(s)$ where $\Delta s = \bar{s} - \underline{s}$ and $E(s) = (\bar{s} + \underline{s})/2$. So Proposition 4 roughly says that when the dispersion in the buyer's valuation is large relative to the expected gain from trade, the seller is more likely to set a minimum price higher than the buyer's minimum valuation. Intuitively, the more uncertain the seller is about the buyer's valuation, the more likely she wants to "over-commit" the delegate in order to ensure a relatively high price in most states of the world. On the other hand, if the expected valuation is high relative to the dispersion of valuation, then the seller does not want to risk losing potential profitable deals by over-committing the delegate. To see this last point, consider the converse of Proposition 4. From Equation (1.10), it is clear that if the valuation distribution satisfies $sg(s)/[1 - G(s)] \geq 1$ for every s, then the seller will always set $z^B = \underline{s}$. For uniformly distributed valuation, this condition simplifies to $2\underline{s} \geq \bar{s}$, or $E(s) \geq 1.5\Delta s$. So when the uncertainty about valuation is relatively small, the seller will set $z^B = \underline{s}$.

The next proposition shows the relationship between the optimal effort and minimum price.

Proposition 5 *In the seller's optimal mechanism, the optimal effort level $e^B(t)$ is non-decreasing in the delegate's type, and the optimal minimum price $z^B(t)$ is non-increasing in the delegate's type. Therefore, higher type delegates are given more chance of success in agreement and work harder than lower types.*

Proof: See the Appendix.

The key to understanding Proposition 5 is that commitment through minimum price and the delegate's effort are *substitutes* for the seller. An easy way to see this is through the bargaining outcome equation $x = r(e)(s - z) + z$. Clearly, the marginal revenue of effort decreases in the minimum price z. More formally, one can see from Equation (1.8) that the seller's expected payoff function $EU_P(e, -z, t)$ is supermodular in $(e, -z, t)$. By the monotone comparative statics (see Milgrom and Shannon [47]), $e^B(t)$ and $-z^B(t)$ must be non-decreasing in t. Intuitively, Proposition 5 says that since it is relatively easier to induce a more able delegate to work hard and get a good price, the seller will impose a smaller minimum price for him to reduce the chance of no deal.

1.3.2 Marketing Effort

Now we suppose that the delegate's effort is spent on marketing to attract or find a buyer. The delegate finds a buyer with probability $p(e) \in (0,1)$, where $p(e) = p_0 + p'e$. For simplicity, the delegate's bargaining power relative to the buyer is assumed to be fixed and equals $r_0 \in (0,1)$.

We still focus on linear contracts with minimum prices as in Equation (1.6). Clearly Lemma 3 from Section 1.3.1 applies here for a constant r_0. But the seller will get a positive price and pay the delegate a commission only when the delegate finds a buyer. It is also easy to see that Lemma 4 holds for marketing effort as well, that is, the seller will set a minimum price no less than $\underline{s}$. Commitment with a minimum price equal to $\underline{s}$ is costless to the seller, so she should take advantage of it. Again the central question is whether the seller wants to set a minimum price above $\underline{s}$. To answer this question we have to analyze the following optimal mechanism problem:

$$\max_{\{\alpha(t),\beta(t),e(t),z(t)\}} EU_P = \int_{\underline{t}}^{\bar{t}} \left\{ p(e) \int_{z(t)}^{\bar{s}} [x - \beta(t)(x - z(t))]\, dG(s) - \alpha(t) \right\} dF(t) \tag{1.13}$$

subject to

(i) $(t, e(t)) \in argmax_{\{\hat{t},e\}}\ U_D = \alpha(\hat{t}) + p(e)\beta(\hat{t}) \int_{z(\hat{t})}^{\bar{s}} (x - z(\hat{t}))dG(s) - C(e,t)$

(ii) $U_D(t) \geq U_0$, $\forall t$

(iii) $x = r_0(s - z) + z$, for $s \geq z$, and 0 otherwise

(iv) $z(t) \in [\underline{s}, \bar{s}]$ for all t

Notice that the delegate's expected payoff is the same as in the case of bargaining effort with $r(e)$ being replaced by $r_0 p(e)$. The only difference with problem (1.7) is how the delegate's effort affects the seller's expected payoff. Bargaining effort increases only the share from the revenue *net* of the minimum price, while marketing effort increases the probability of getting a certain amount of revenue *including the minimum price*.

As before, this problem can be solved in two steps. First we find the conditions for the optimal effort $e^M(t)$ and minimum price $z^M(t)$ (where M stands for "marketing"). Using the same technical steps as in the proof of Proposition 1, we can rewrite the problem as:

$$\max_{\{e(t),z(t)\}} \int_{\underline{t}}^{\bar{t}} \left\{ p(e) \Big[r_0 E[s-z|s \geq z] + z[1-G(z)] \Big] - C(e,t) + C_t(e,t)[\frac{1-F(t)}{f(t)}] \right\} dF(t) - U_0 \tag{1.14}$$

Let $e^M(t)$ and $z^M(t)$ be the level of effort and minimum price that solve this problem. We assume interior solution for z^M. By point-wise differentiation of Equation (1.14), $e^M(t)$ and $z^M(t)$ must satisfy the following first-order conditions:

$$p' \Big\{ r_0 E[s-z^M|s \geq z^M] + z^M[1-G(z^M)] \Big\} = C_e(e^M,t) - [\frac{1-F(t)}{f(t)}] C_{et}(e^M,t) \tag{1.15}$$

$$p(e^M) \Big[(1-r_0)(1-G(z^M)) - z^M g(z^M) \Big] = 0 \tag{1.16}$$

The second step is to find the contract coefficients α^M and β^M that satisfy the IC and participation constraints and that implement the optimal effort e^M. Let $\alpha^M(\hat{t})$ and $\beta^M(\hat{t})$ in contract (1.6) be such that:

$$\alpha^M(\hat{t}) = C(e^M(\hat{t}),\hat{t}) - \int_{\underline{t}}^{\hat{t}} C_t(e^M(\nu),\nu)d\nu - \frac{C_e(e^M(\hat{t}),\hat{t})}{p'} p(e^M(\hat{t})) + U_0$$
$$\beta^M(\hat{t}) = \frac{C_e(e^M(\hat{t}),\hat{t})}{r_0 p' E[s - z^M(\hat{t}) | s \geq z^M(\hat{t})]} \tag{1.17}$$

The next proposition states that the contract (1.17) together with the minimum price $z^M(t)$ induces the delegate to exert the recommended effort $e^M(t)$.

Proposition 6 *The linear contract (1.17) with the optimal minimum price $z^M(t)$ implements the recommended effort $e^M(t)$ and induces the delegate to report his true type.*

Proof: See the Appendix.

Comparing Equations (1.10) and (1.16), one can see that the first-order conditions for the optimal minimum price are very similar in the two cases of bargaining and marketing effort. The main difference is that in the case of marketing effort, the minimum price can be solved from Equation (1.16) alone, and only depends on the distribution of the buyers' valuations and the delegate's relative bargaining power but not on the delegate's effort. The minimum price is also independent of the delegate's type. Similar to Proposition 4, we have the following result:

Proposition 7 *If $1 - r_0 > \underline{s}g(\underline{s})$, then the seller will set a minimum price above $\underline{s}$ for every delegate. Thus, with positive probability, the delegate and the buyer will not make a deal.*

This proposition says that, as in the case of bargaining effort, the seller's strategic manipulation of the delegation contract may cause bargaining failures between the delegate and the buyer. It is easy to see that when the buyer's valuation is disperse relative to the expected gains of trade, then the seller's optimal minimum price will be more likely to exceed the buyer's lowest valuation.

Unlike in the case of bargaining effort, commitment through minimum prices and incentives are *no longer substitutes* with marketing effort. In fact, since the minimum price can be solved from Equation (1.16) alone, the minimum price and effort do not display either substitute or complementary relations.

1.4 Discussions

1.4.1 Comparison with Other Trading Mechanisms

Hiring a delegate to bargain with consumers is one of the trading mechanisms used in real life. Another commonly used mechanism is posted-price selling, whereby the seller commits to a fixed price. The optimal fixed price for the seller, τ, maximizes the expected profit

$$\max_{\{\tau\}} EU_P = \int_{\tau}^{\bar{s}} \tau dG(s) = \tau[1 - G(\tau)]$$

So τ is given by

$$1 - G(\tau) = \tau g(\tau) \tag{1.18}$$

This is the standard monopoly pricing formula. Comparing it with Equations (1.10) and (1.16), one finds that the optimal posted price τ is greater than the minimum prices as long as the delegate's bargaining power (measured by his share r)

is positive. And the difference between the optimal posted price τ and the optimal minimum price increases in the delegate's bargaining power. When the delegate has zero bargaining power ($r = 0$), then the minimum price coincides with the optimal posted price. In this case, the delegate does not bring in additional sales revenue to the seller. On the other hand, when the delegate's bargaining power is very large (r approaches one), the optimal minimum price goes to $\underline{s}$, and trade is almost efficient. In this case, the final sales price is close to the buyer's valuation. So the outcome resembles a perfectly discriminating monopolist.

In general cases where the delegate has positive but not full bargaining power, the trade outcome falls in between posted-price selling and perfectly discriminating monopoly pricing. Perfectly discriminating monopoly pricing requires that the monopolist knows every buyer's valuation and can commit to a take-it-or-leave-it price offer to buyers. When the buyer's valuation is not observed, what we study is an alternative to posted-price selling, namely, the seller hires a delegate to find out the buyer's valuation and bargain over a price. This trading mechanism removes some of the rigidity in posted-price selling, and thus the use of the delegate improves trade efficiency. Since the minimum price decreases in the delegate's bargaining power, the efficiency gain associated with the use of delegate increases in his bargaining power. Of course, whether the seller gets more profit by hiring the delegate relative to posted-price selling also depends on the delegate's bargaining power, the cost of hiring him, and the distribution of the buyer's valuation.[10]

1.4.2 Commitment Power of Agency Contracts

A critical question in the delegation literature is whether and when a delegation contract can be credibly used as a commitment device. Here we briefly discuss some of the factors that may affect the credibility of the contracts studied in the preceding section as a commitment device.

Recall that the commitment effect in our model comes from the minimum price only. As Lemma 3 shows, neither the fixed wage nor the commission rate of the delegate's compensation contract affects the bargaining outcome. So the key to the question of credibility is whether the buyer can be convinced that the minimum price is indeed the limit of the delegate's discretion over price. How can the buyer be sure that the delegate is not lying about the minimum price? What is to prevent the delegate and the seller from rescinding the minimum price, especially when the buyer's valuation is just below it?

Not Perfectly Observable Contract. If the delegation contract is not perfectly observable to the buyer, the delegate may have a tendency to claim that a minimum price close to the buyer's valuation is set by the seller. But this tendency may destroy the credibility of using the minimum price as a commitment device. So whether

[10] Wang [60] compares seller self-bargaining with posted-price selling in a different model.

unobservable delegation contracts have any commitment value in our model is not clear. A thorough analysis would start by specifying the buyer's prior belief about the minimum price (whether it is set, and how much it is) and then study the bargaining game with such asymmetric information on the buyer's part. This is beyond the scope of this work. But if we still assume that the delegate knows the buyer's valuation, we can use the results from Gul, Sonnenschein and Wilson [26] and Gul and Sonnenschein [25] to show that under fairly reasonable conditions the minimum prices can be revealed rather quickly in equilibrium, which implies that unobservable minimum prices still have considerable commitment power. The main problem, however, is that such bargaining models under asymmetric information often yield multiple equilibria. Moreover, if the delegate does not perfectly know the buyer's valuation, things become completely intractable.

As mentioned before, Katz [32], Fershtman and Kalai [20], Corts and Neher [10], Kockesen and Ok [37], and many others have addressed the issue of whether unobservable contracts can still serve as a credible commitment device. Depending on the other party's belief, these papers find that unobservable contracts can still have commitment value when some equilibrium refinements are used. This suggests that our results are valid to some extent even when minimum prices are not perfectly observed by the buyers.

Renegotiable Delegation Contracts. A related credibility issue arises if the seller and the delegate can renegotiate the delegation contract.[11] In our model, renegotiation can be especially relevant when the delegate finds out that the buyer's valuation is below the minimum price. A Pareto improvement is readily available if the minimum price in the delegate's compensation contract is lowered. But if renegotiation is possible in such cases, then there is no strong reason why it cannot be in any other cases.

To maintain the credibility of the minimum price, the seller and the delegate may rely on reputation effects (see discussions below) or other sorts of institutions. In the case of car dealership, the dealer invoice price (and other related contractual provisions between car manufactures and car dealers) can be thought of an institutional innovation to maintain credibility with the help of car manufacturers.[12]

Reputation in Multi-Unit Sale. In situations such as car dealerships, the delegate is hired to sell same products over time. Our model and our results extend easily to the case in which there are N potential buyers with identical and independent distribution of valuations. On the credibility issue (which our model does not directly address), repeated sales may make it easier for the seller and the delegate to commit to a minimum price than a single-unit sale. When information about prices from

[11] See, e.g., Dewatripont [13] and Caillaud *et al.* [8] for analysis of commitment effect when delegation contracts are renegotiable.

[12] Alternatively, car manufactures would simply sell cars to car dealerships at lower prices. It seems difficult to justify going through all the troubles of those contractual provisions (e.g., holdbacks and other incentives) if not for commitment purposes. Contracts between car makers and dealers are franchise contracts, and there are many other important considerations (e.g., competition among dealers), see, e.g., Klein and Murphy [36], Klein [35] and Tirole [57]. See also Bresnahan and Reiss [6] for an early empirical work on pricing practices between car manufacturers and dealers.

past trades is available to later buyers (e.g., through word of mouth communication or consumer reports), the delegate and the seller may have incentives to stick to the minimum price despite short-term gains from trading with low-valuation buyers.

Reputation with Multiple Delegates. Similar reputation effects can arise when the seller hires multiple delegates to conduct sales (e.g., one person owns several dealerships, each of which is run by a manager). If the seller renegotiates with one manager, then it is hard not to renegotiate with other managers, which can reduce the total profit for the seller.

1.5 Comparative Statics and Numerical Examples

In this section we want to derive comparative statics of the model that may be useful in certain applications. In doing so, we need to specify the model a little more further. Specifically, suppose the buyer's valuation is uniformly distributed in $[\underline{s}, \bar{s}]$ and the delegate's type is uniformly distributed in $[\underline{t}, \bar{t}]$. The revenue share the delegate can get is given by $r(e) = r_0 + r'e$ in the case of bargaining effort. The parameter r_0 is the share the delegate can get without extra unobservable effort, and the parameter r' measures how productive the delegate's bargaining effort is (marginal revenue of effort equals $r'E(s)$). To ensure $r(e) \leq 1$, the meaningful range for bargaining effort is constrained to $[0, (1 - r_0)/r']$. On the other hand, in the case of marketing effort, the revenue share the delegate can get is a constant r_0, and the probability of finding a buyer is $p(e) = p_0 + p'e$. In this case, the effort is constrained to $e \leq (1 - p_0)/p'$ to ensure that $p(e) \leq 1$. The delegate's cost function is: $C(e, t) = \gamma_1(\bar{t} - t)e + \gamma_2 e^2$, with γ_1 and γ_2 both positive constants. Finally, we let $U_0 = 0$.

For concreteness, we will solve the model numerically with the following parameter values. The buyer's valuation is uniform on $[10, 950]$, and the delegate's type is uniform on $[0, 1]$. In the bargaining effort case, the delegate's bargaining share is $r(e) = 0.3 + 0.1e$, and $e \in [0, 7]$. In the marketing effort case, the bargaining share is $r_0 = 0.5$. The probability function is $p(e) = 0.3 + 0.1e$, and $e \in [0, 7]$. Under both interpretations, the effort cost function is $C(e, t) = 8(1 - t)e + 12e^2$. In this case, the total expected surplus from trade is 480.

1.5.1 No Commitment Effect

If delegation contracts do not have any commitment effect, our analysis in Section 1.2 shows that the seller's optimal effort schedule should maximize

$$(r_0 + r'e)E(s) - 2\gamma_1(\bar{t} - t)e - \gamma_2 e^2$$

where $E(s) = (\bar{s} + \underline{s})/2$. From Equations (1.4) and (1.5), the seller's desired level of effort and the commission rate of the delegation contract can be easily found as

$$\begin{aligned} e^* &= \frac{r'E(s)}{2\gamma_2} - \frac{\gamma_1(\bar{t} - t)}{\gamma_2} \\ \beta^* &= 1 - \frac{\gamma_1(\bar{t} - t)}{r'E(s)} \end{aligned}$$

The solution to our numerical model is given in Table 1.4.[13] In this case, since the delegate and the buyer will always make a deal, the total expected surplus from trade is 480, which is shared by the seller, the delegate and the buyer. The seller obtains an expected surplus of 169.6, and the buyer gets an expected surplus of 256. The remainder is the delegate's expected wage payment of 54.4, of which 40 is his expected effort cost and 14.4 his expected information rent.

The comparative statics are straightforward and are summarized in Table 1.1.

Table 1.1: Comparative Statics: No Commitment

Increase in	t	$\bar{t}$	r'	$E(s)$	γ_1	γ_2
e^*	↑	↓	↑	↑	↓	↓
β^*	↑	↓	↑	↑	↓	–

These results are easy to understand. Since higher type delegates have lower marginal effort costs, optimal effort (and hence incentives through commission rate) should increase in type. The marginal revenue of effort is the product of r' and the expected total surplus $E(s)$. Hence, holding other things fixed, increase in r' or the expected total surplus will lead to higher commission rates and greater effort. The parameter r' measures the importance of effort. When $r' = 0$, the moral hazard problem disappears. In this case, $\beta^* = 0$ and $e^* = 0$, and the seller pays the delegate a fixed wage equal to his reservation utility. On the other hand, the parameter γ_2

[13] The detailed solutions to the numerical model are presented in several Tables at the end of the chapter.

measures the difficulty of inducing high effort for any given type of delegate, hence has the opposite effect on the optimal effort as r'. The commission rate β^* is independent of γ_2 because the "physical" effort cost $\gamma_2 e^2$ is compensated by the fixed payment α^*.

Holding other things fixed, increase in $\bar{t}$ means that the degree of adverse selection is greater between the seller and the delegate and hence makes it harder to induce truth-telling from the delegate. Consequently, ceteris paribus, the higher $\bar{t}$, the lower the optimal effort and commission rate. To see this more clearly, consider the extreme case in which $\bar{t}$ collapses to $\underline{t}$ so that there is no adverse selection. Then $t = \bar{t} = \underline{t}$, so $\beta^* = 1$ and $e^* = r'E(s)/(2\gamma_2) = e_{FB}$. This is simply the standard result that the efficient outcome (for the seller) can be achieved with a sell out contract when there is moral hazard and the agent is risk-neutral.

The parameter γ_1 measures the intensity of agency problem between the seller and the delegate. Higher γ_1 means that different delegates differ more in their dislike of effort, which leads to higher information rents. Consequently, other things being equal, the seller would want to set a higher commission rate and induce greater effort from the delegate when γ_1 is lower. When $\gamma_1 = 0$, the delegate's type does not matter, and the seller should sell the good to the delegate.

1.5.2 Bargaining Effort

Now suppose delegation contracts have commitment power and the delegate exerts bargaining effort. From Section 1.3.1, for every type t, the seller's desired effort and minimum price should maximize

$$\begin{aligned} U_{P,t} &= (r_0 + r'e)\frac{(\bar{s}-z)^2}{2\Delta s} + z\frac{\bar{s}-z}{\Delta s} - 2\gamma_1(\bar{t}-t)e - \gamma_2 e^2 \\ &= (r_0 + r'e)\frac{(E(s)+\frac{\Delta s}{2}-z)^2}{2\Delta s} + z\frac{(E(s)+\frac{\Delta s}{2}-z)}{\Delta s} - 2\gamma_1(\bar{t}-t)e - \gamma_2 e^2 \end{aligned}$$

We write $\bar{s} = E(s) + \Delta s/2$, where $\Delta s = \bar{s} - \underline{s}$, because we would like to separate out the effects of changes in the expected surplus and changes in the uncertainty (dispersion) of valuations.

The closed form solutions for the optimal effort and minimum price are not readily available from the first-order conditions (which lead to cubic equations of e and z). Table 1.5 gives the solution for our numerical example. In this case, the optimal effort is much lower than the case with no commitment. Moreover, the optimal minimum price is set in between $[365, 390]$. This implies that the chance of bargaining failure is about 39 %. Because of the commitment effect, the seller's expected payoff jumps to 281.33, more than 65 % higher than that in the case of no commitment effect. The delegate's effort cost and information rent are both much lower. The buyer is also screwed, getting an expected payoff of 115, which is less than half of that in the case of no commitment. Bargaining failures cause welfare loss of about 76, about 16 % of the total expected surplus.

We derive comparative statics results for the case of bargaining effort (details in the Appendix), which are summarized in Table 1.2 below.

The comparative statics of e^B and β^B with respect to $\{t, \bar{t}, r', E(s), \gamma_1, \gamma_2\}$ are the same as in the case with no commitment effect, and have the same interpretations as

Table 1.2: Comparative Statics: Bargaining Effort

Increase in	t	$\bar{t}$	r'	$E(s)$	Δs	γ_1	γ_2
e^B	↑	↓	↑	↑	↓	↓	↓
z^B	↓	↑	↓	↑/↓	↑	↑	↑
β^B	↑	↓	↑	↑	↓	↓	↓

The cells with two arrows indicate ambiguous comparative statics.

given in the previous subsection.

A new implication from commitment effect is that now the delegate's optimal effort and his incentives (measured by β^B) are lower if uncertainty about the buyer's valuation increases (i.e., Δs increases while holding $E(s)$ fixed). Without commitment effect, uncertainty about the buyer's valuation does not matter because both the seller and the delegate are risk-neutral. With commitment effect, the seller sets a minimum price z^B that can be higher than the buyer's lowest valuation. When $E(s)$ is fixed and the dispersion of valuation Δs increases, the buyer's lowest valuation must decrease. It follows that more likely the buyer's valuation falls below a fixed minimum price. Moreover, the optimal minimum price will increase in this case (see below). Thus, the probability of bargaining failure increases. As a result, the expected return to bargaining effort is reduced, thus leading to lower effort and lower incentives.

Another set of comparative statics results in Table 1.2 concerns the minimum price. In Section 1.3.1 we show that incentives and minimum prices are substitutes (Proposition 5) and they move in opposite directions as the delegate's type changes.

In fact, this is also true with respect to $\bar{t}$, r', γ_1 and γ_2 (see the Appendix). Basically, when agency problems are more severe and hence it is more costly to induce efforts (higher $\bar{t}$, γ_1 and γ_2), then the seller will substitute incentives for commitment by increasing the minimum price. On the other hand, when effort is more productive (higher r'), then the seller will reduce the minimum price. When the expected surplus $E(s)$ increases (holding Δs fixed), there are two opposite effects on the minimum price. On one hand, effort is more productive, hence the minimum price should go down. On the other hand, since both $\underline{s}$ and $\bar{s}$ increase, the cost of commitment (i.e., no deal) decreases while the benefit of commitment increases, so the minimum price should go up. The net effect of $E(s)$ on z is thus ambiguous. For example, if effort is not very productive (low r') or is costly (high γ_2) or uncertainty about valuation Δs is relatively high, then the second effect dominates so the minimum price increases in $E(s)$. When the dispersion of valuation Δs increases (holding $E(s)$ fixed), the marginal cost of using minimum prices becomes relatively smaller than the marginal benefit. Therefore, minimum price increases in Δs. Moreover, effort will go down, also leading to higher minimum price.

1.5.3 Marketing Effort

Now we turn to the case of marketing effort. From Section 1.3.2, for every type t, the seller's desired effort and minimum price should maximize

$$U_{P,t} = (p_0 + p'e)\left[r_0 \frac{(E(s) + \frac{\Delta s}{2} - z)^2}{2\Delta s} + z\frac{(E(s) + \frac{\Delta s}{2} - z)}{\Delta s}\right] - 2\gamma_1(\bar{t} - t)e - \gamma_2 e^2$$

It is easy to check that the optimal effort, the minimum price and the slope of the contract are given by

$$
\begin{aligned}
e^M &= \frac{p'[E(s)+\frac{\Delta s}{2}]^2}{4\gamma_2(2-r_0)\Delta s} - \frac{\gamma_1(\bar{t}-t)}{\gamma_2} \\
z^M &= \frac{1-r_0}{2-r_0}[E(s)+\frac{\Delta s}{2}] \\
\beta^M &= \frac{2-r_0}{r_0} - \frac{2\gamma_1(\bar{t}-t)(2-r_0)^2\Delta s}{r_0 p'[E(s)+\frac{\Delta s}{2}]^2}
\end{aligned}
$$

The solution to our numerical example is presented in Tables 1.6 (no commitment) and 1.7 (commitment).

Unlike the bargaining effort case, now the optimal effort is higher with commitment than without commitment, that is, incentives and commitment are complements in this example. Consequently, the probability of finding a buyer is higher with commitment, and the commission rate is much higher (greater than 2), which resembles the results in Fershtman and Judd [18, 17] that managers are "over-compensated" on the margin in equilibrium. The seller imposes a minimum price of 316, resulting in a 33% chance of bargaining failure. The welfare loss from bargaining failures is about 11% if holding effort fixed, about 3% if compared to the case under no commitment. The seller's expected utility increases from 74 to 103 (around 40%) due to both the commitment and incentive effects. The buyer is again the victim of the seller's commitment scheme, seeing his expected utility plunge from 88 to 43.

The comparative statics are summarized in Table 1.3.

The comparative statics of effort e^M and commission rate β^M are basically the same as in the case of bargaining effort. The minimum price now is independent of all

Table 1.3: Comparative Statics: Marketing Effort

Increase in	t	$\bar{t}$	p'	$E(s)$	Δs	γ_1	γ_2	
e^M	↑	↓	↑	↑	↓	↓	↓	
z^M	–	–	–	↑	↑	–	–	
β^M	↑	↓	↑	↑	↓	↓	–	

the variables except the buyer's valuations (and the delegate's bargaining power r_0). Furthermore, the minimum price and effort are positively related when the expected valuation changes, but negatively related when the dispersion of valuation increases.

1.6 Conclusion

We develop a framework that can be used to analyze the interactions between agency problems and commitment effect in delegated bargaining situations. We find that the seller's strategic manipulation of the delegation contracts can cause bargaining failures between her delegate and the buyer. Furthermore, the interactions between incentives and commitment depend on the nature of the agency problem: they are substitutes in the case of bargaining effort but not in the case of marketing effort. We also derive comparative statics of the model, some of which may possibly lead to testable implications. Empirical work is badly needed for the delegation literature, because, to our best knowledge, there has been no empirical study providing evidence on the existence of strategic delegation despite a large number of theoretical works.

1.7 Appendix to Chapter 1

Proof of Proposition 1: Let EU_P be the seller's expected utility when she pays the delegate a wage $w(\hat{t}, x)$, that is,

$$EU_P = \int_{\underline{t}}^{\bar{t}} E[x - w(\hat{t}, x)]dF(t) = \int_{\underline{t}}^{\bar{t}} \left\{r(e)E(s) - E[w(\hat{t}, x)]\right\} dF(t) \tag{1.19}$$

and let $U_D(\hat{t}, t)$ be the type-t delegate's utility when he announces type $\hat{t}$, which is

$$U_D(\hat{t}, t) = E[w(\hat{t}, x)] - C(e, t) \tag{1.20}$$

where the expectation $E[.]$ in these two equations is taken over the random variable s.

Consider a seller's effort recommendation $e(\hat{t})$. Suppose the delegate follows it. The IC condition reduces to truth-telling only. The first-order condition with respect to the delegate's type announcement is

$$\frac{\partial U_D(\hat{t}, t)}{\partial \hat{t}}|_{\hat{t}=t} = 0$$

Let $U_D(t) = U_D(t, t)$ be the delegate's utility when he reports his true type. The total derivative of $U_D(t)$ with respect to his type report can be obtained from the Envelope Theorem as follows

$$\frac{dU_D(\hat{t}, t)}{dt}|_{\hat{t}=t} = \frac{\partial U_D(\hat{t}, t)}{\partial \hat{t}}|_{\hat{t}=t} + \frac{\partial U_D(\hat{t}, t)}{\partial t}|_{\hat{t}=t} = \frac{\partial U_D(\hat{t}, t)}{\partial t}|_{\hat{t}=t} = -C_t(e, t)$$

where the last equality comes from Equation (1.20). Since this is a total derivative the delegate's utility can be reconstructed by integrating this equation with respect to his type.

$$U_D(t) = U_D(\underline{t}) - \int_{\underline{t}}^{t} C_t(e, \nu)dF(\nu) \tag{1.21}$$

So, from Equations (1.20) (evaluated at the delegate's true type) and (1.21) we can solve for the wage schedule as follows

$$E[w(t, x)] = U_D(t) + C(e, t) = U_D(\underline{t}) - \int_{\underline{t}}^{t} C_t(e, \nu)dF(\nu) + C(e, t)$$

Plugging the wage schedule into the seller's expected utility function (Equation (1.19)) gives

$$EU_P = \int_{\underline{t}}^{\bar{t}} \left\{r(e)E(s) - C(e, t) + \int_{\underline{t}}^{t} C_t(e, \nu)dF(\nu)\right\} dF(t) - U_D(\underline{t}) \tag{1.22}$$

Next, integrating by parts the second term of the integral yields

$$\int_{\underline{t}}^{\bar{t}}\int_{\underline{t}}^{t} C_t(e,\nu)dF(\nu)dF(t) =$$

$$= \left[-(1-F(t))\int_{\underline{t}}^{t} C_t(e,\nu)dF(\nu)\right]_{\underline{t}}^{\bar{t}} + \int_{\underline{t}}^{\bar{t}}[\frac{1-F(t)}{f(t)}]C_t(e,t)dF(t) =$$

$$= \int_{\underline{t}}^{\bar{t}}[\frac{1-F(t)}{f(t)}]C_t(e,t)dF(t) \quad (1.23)$$

Note that if the seller ensures a type-$\underline{t}$ delegate a utility $U_D(\underline{t}) = U_0$, the interim participation constraint is satisfied for all types. The reason is that the delegate's expected utility function (Equation (1.21)) is increasing in t since C_t is negative. Hence the seller should set $U_D(\underline{t}) = U_0$.

Using Equation (1.23) and $U_D(\underline{t}) = U_0$, one can rewrite Equation (1.22) as

$$EU_P = \int_{\underline{t}}^{\bar{t}} \left\{ r(e)E(s) - C(e,t) + C_t(e,t)\frac{1-F(t)}{f(t)} \right\} dF(t) - U_0$$

This is Equation (1.3). Note that the seller has to pay the delegate his effort cost, his reservation utility and some information rent. The seller will choose an effort recommendation that maximizes her expected payoff. Differentiating point-wise with respect to effort, we get the following first-order condition for $e^*(t)$:

$$r'E(s) - C_e(e^*,t) + \frac{1-F(t)}{f(t)}C_{et} = 0$$

This is Equation (1.4). The second-order condition is clearly satisfied because the integrand in Equation (1.3) is concave in e: r' is a constant, $C_{ee}(.,t) > 0$, and C_{et} is a constant. *Q.E.D.*

Proof of Proposition 2: For later reference, notice that effort $e^*(t)$ is non-decreasing in type. From Equation (1.4), it is clear that the "total" marginal cost of effort decreases with type (the inverse of the hazard rate decreases with type and C_{et} is negative). By the monotone comparative statics (Milgrom and Shannon [47]), effort must be non-decreasing in t.

If the seller offers the delegate the contract (1.5), the delegate's utility when he exerts effort e and reports $\hat{t}$ is

$$U_D(\hat{t},e,t) \;=\; C(e^*(\hat{t}),\hat{t}) + \frac{C_e(e^*(\hat{t}),\hat{t})}{r'}[r(e) - r(e^*(\hat{t}))] -$$

$$-\int_{\underline{t}}^{\hat{t}} C_t(e^*(\nu),\nu)d\nu - C(e,t) + U_0 \tag{1.24}$$

The first-order conditions are the following:

$$\begin{aligned}\frac{\partial U_D(\hat{t},e,t)}{\partial e} &= C_e(e^*(\hat{t}),\hat{t}) - C_e(e,t) = 0\\ \frac{\partial U_D(\hat{t},e,t)}{\partial \hat{t}} &= \frac{[r(e)-r(e^*(\hat{t}))]}{r'}\frac{d}{d\hat{t}}C_e(e^*(\hat{t}),\hat{t}) = 0\end{aligned}$$

They are satisfied at $\hat{t} = t$ and $e = e^*(t)$. The second-order conditions for a maximum are also satisfied since the delegate's profit is concave in effort and the determinant of the second-order matrix is positive.

$$\frac{\partial^2 U_D(t,e,t)}{\partial e^2}\frac{\partial^2 U_D(t,e,t)}{\partial t^2} - \left(\frac{\partial^2 U_D(t,e,t)}{\partial e \partial t}\right)^2 = -C_{et}\frac{d}{dt}C_e(e^*(t),t) \geq 0$$

This last inequality holds because of the following equation (derived from Equation (1.4)):

$$\frac{d}{dt}C_e(e^*(t),t) = C_{et}\frac{\partial}{\partial t}[\frac{1-F(t)}{f(t)}] \geq 0 \tag{1.25}$$

Q.E.D.

Proof of Corollary 1: That $\beta^*(t)$ is non-decreasing in type can be checked from Equation (1.25). In the beginning of the proof of Proposition 2 we showed that the effort is non-decreasing in type. From the definition of $\alpha^*(t)$ (Equation (1.5)),

$$\frac{\partial \alpha^*(t)}{\partial t} = -\frac{r(e^*(t))}{r'}\frac{d}{dt}C_e(e^*(t),t) \leq 0$$

Q.E.D.

Proof of Lemma 4: Consider any direct revelation mechanism ($\alpha(\hat{t})$, $\beta(\hat{t})$, $z(\hat{t})$, $e(\hat{t})$), where $z(\hat{t}) < \underline{s}$. This mechanism gives the seller a revenue of $r(e(t))[E(s) - z(t)] + z(t)$. But the seller can do better with another mechanism which also implement the same effort recommendation $e(\hat{t})$ but imposes the minimum price equal to $\underline{s}$. Consider the following mechanism ($\tilde{\alpha}(\hat{t})$, $\beta(\hat{t})$, $\underline{s}$, $e(\hat{t})$), where $\tilde{\alpha}(\hat{t}) = \alpha(\hat{t}) + \beta(\hat{t})r(e(\hat{t}))(\underline{s} - z(\hat{t}))$. The expected wage is the same since

$$E[w(x,\hat{t})] \;=\; \tilde{\alpha}(\hat{t}) + \beta(\hat{t})E[x - \underline{s}] =$$

$$
\begin{aligned}
&= \tilde{\alpha}(\hat{t}) + \beta(\hat{t})r(e(\hat{t}))[E(s) - \underline{s}] = \\
&= \alpha(\hat{t}) + \beta(\hat{t})r(e(\hat{t}))(\underline{s} - z(\hat{t})) + \beta(\hat{t})r(e(\hat{t}))[E(s) - \underline{s}] = \\
&= \alpha(\hat{t}) + \beta(\hat{t})r(e(\hat{t}))[E(s) - z(\hat{t})]
\end{aligned}
$$

All the (IC) and (IR) conditions must be satisfied as they are in the old mechanism $(\alpha(\hat{t}), \beta(\hat{t}), z(\hat{t}), e(\hat{t}))$. The cost to the seller is also the same, but her expected revenue increases since

$$
\begin{aligned}
E(x) &= r(e(t))E[s - \underline{s}] + \underline{s} = r(e(t))E(s) + [1 - r(e(t))]\underline{s} > \\
&> r(e(t))E(s) + [1 - r(e(t))]z(t) = r(e(t))[E(s) - z(t)] + z(t)
\end{aligned}
$$

Q.E.D.

Proof of Proposition 3: The type-t delegate's utility when he reports $\hat{t}$, chooses e and is paid according to contract (1.11) is

$$
\begin{aligned}
U_D(\hat{t}, e, t) &= \alpha^B(\hat{t}) + \beta^B(\hat{t})r(e)E[s - z^B(\hat{t})|s \geq z^B(\hat{t})] - C(e, t) \\
&= C(e^B(\hat{t}), \hat{t}) + \frac{C_e(e^B(\hat{t}), \hat{t})}{r'}[r(e) - r(e^B(\hat{t}))] - \int_{\underline{t}}^{\hat{t}} C_t(e^B(\nu), \nu)d\nu - C(e, t) + U_0
\end{aligned}
$$

Notice the similarity between this utility and that of Equation (1.24). The proof is similar to that of Proposition 2 with a change of the superscript "$*$" to the superscript "B" and a change of $E(s)$ to $E[s - z(\hat{t})|s \geq z(\hat{t})]$. The second-order condition is satisfied because $C_e(e^B(t), t)$ is non-decreasing in type. From Equation (1.9),

$$
\begin{aligned}
\frac{d}{dt}C_e(e^B(t), t) &= r'\frac{\partial E[s - z^B|s \geq z^B]}{\partial z}\frac{\partial z^B}{\partial t} + C_{et}\frac{d}{dt}\frac{1 - F(t)}{f(t)} = \\
&= -r'[1 - G(z)]\frac{\partial z}{\partial t} + C_{et}\frac{\partial}{\partial t}\frac{1 - F(t)}{f(t)}
\end{aligned}
$$

Proposition 5 shows that z is non-increasing in type. Hence the first term of the equality is non-negative. From Equation (1.25), the second term is also non-negative.

Q.E.D.

Proof of Proposition 5: From the seller's expected payoff function in Equation (1.8), we can show that

$$\begin{aligned}
\frac{\partial^2 EU_P}{\partial e \partial(-z)} &= r' \frac{\partial E[s-z|s \geq z]}{\partial(-z)} = r'[1-G(z)] \geq 0 \\
\frac{\partial^2 EU_P}{\partial e \partial t} &= -C_{et}\left(1 - \frac{\partial}{\partial t}\frac{1-F(t)}{f(t)}\right) \geq 0 \\
\frac{\partial^2 EU_P}{\partial(-z)\partial t} &= 0
\end{aligned}$$

Therefore, $EU_P(e,-z,t)$ is supermodular, and by the monotone comparative statics, $e(t)$ is non-decreasing in t and z is non-increasing in t. *Q.E.D.*

Proof of Proposition 6: Recall that $p(e)$ in Section 1.3.2 has the same interpretation as $r(e)$ in Sections 1.2 and 1.3.1. The delegate's utility under contract (1.17) is

$$\begin{aligned}
U_D(\hat{t},e,t) &= \alpha^M(\hat{t}) + \beta^M(\hat{t}) r_0 p(e) E[s - z^M(\hat{t})|s \geq z^M(\hat{t})] - C(e,t) = \\
&= C(e^M(\hat{t}),\hat{t}) + \frac{C_e(e^M(\hat{t}),\hat{t})}{p'}[p(e) - p(e^M(\hat{t}))] - \int_{\underline{t}}^{\hat{t}} C_t(e^M(\nu),\nu)d\nu - C(e,t) + U_0
\end{aligned}$$

Next compare the delegate's utility under this contract with his utility in the proof of Proposition 2 (see Equation (1.24)). The rest of the proof is similar to that of Proposition 2 with a change of the superscript "$*$" to the superscript "M". The second-order condition is satisfied because $C_e(e^M(t),t)$ is non-decreasing with type. Using Equation (1.15), and taking into account that z^M does not change with type (from Equation (1.16)),

$$\frac{d}{dt}C_e(e^M(t),t) = C_{et}\frac{\partial}{\partial t}[\frac{1-F(t)}{f(t)}] \geq 0$$

Q.E.D.

Comparative statics: Bargaining Effort (Table 1.2): The seller's utility for a given type t, assuming that the parameters are such that $z^B \in (\underline{s}, \bar{s})$, is

$$U_{P,t} = (r_0 + r'e)\frac{(E(s) + \frac{\Delta s}{2} - z)^2}{2\Delta s} + z\frac{(E(s) + \frac{\Delta s}{2} - z)}{\Delta s} - 2\gamma_1(\bar{t} - t)e - \gamma_2 e^2$$

This function, written as $U_{P,t}(e,-z,t,-\bar{t},r',-\gamma_1,-\gamma_2)$, is supermodular since

$$\begin{aligned}
\frac{\partial^2 EU_{P,t}}{\partial e \partial z} &= -r'\frac{(E(s)+\frac{\Delta s}{2}-z)}{\Delta s} < 0 \qquad \frac{\partial^2 EU_{P,t}}{\partial e \partial t} = 2\gamma_1 > 0 \qquad \frac{\partial^2 EU_{P,t}}{\partial e \partial \bar{t}} = -2\gamma_1 < 0 \\
\frac{\partial^2 EU_{P,t}}{\partial e \partial r'} &= \frac{(E(s)+\frac{\Delta s}{2}-z)^2}{2\Delta s} > 0 \qquad \frac{\partial^2 EU_{P,t}}{\partial e \partial \gamma_1} = -2(\bar{t}-t) \leq 0 \qquad \frac{\partial^2 EU_{P,t}}{\partial e \partial \gamma_2} = -2e < 0 \\
\frac{\partial^2 EU_{P,t}}{\partial z \partial r'} &= -e\frac{(E(s)+\frac{\Delta s}{2}-z)}{\Delta s} < 0 \qquad \frac{\partial^2 EU_{P,t}}{\partial z \partial t} = \frac{\partial^2 EU_{P,t}}{\partial z \partial \bar{t}} = \frac{\partial^2 EU_{P,t}}{\partial z \partial \gamma_1} = \frac{\partial^2 EU_{P,t}}{\partial z \partial \gamma_2} = 0
\end{aligned}$$

By the monotone comparative statics, e and $-z$ are non-decreasing in t and r' and non-increasing in $\bar{t}$, γ_1 and γ_2.

The equation for the commission β^B is given by

$$\beta^B = 1 - \frac{2\gamma_1(\bar{t}-t)(2-r_0-r'e)^2\Delta s}{r'[E(s)+\frac{\Delta s}{2}]^2} \tag{1.26}$$

This commission increases in t and r', and decreases in $\bar{t}$, γ_1 and γ_2.

The response of effort and minimum price to changes in $E(s)$ and Δs is not straightforward, but we can get some results from the first-order conditions. Combining those two conditions ((1.9) and (1.10)) we obtain the following equations (they are displayed in Figures 1.1 and 1.2):

$$\frac{r'[E(s)+\frac{\Delta s}{2}]^2}{2\Delta s(2-r_0-r'e)^2} = 2\gamma_1(\bar{t}-t) + 2\gamma_2 e \tag{1.27}$$

$$z = \left[1 - r_0 - r'\left\{\frac{r'(E(s)+\frac{\Delta s}{2}-z)^2}{4\gamma_2\Delta s} - \frac{\gamma_1(\bar{t}-t)}{\gamma_2}\right\}\right](E(s)+\frac{\Delta s}{2}-z) \tag{1.28}$$

We can see that the left-hand side of Equation (1.27) increases in $E(s)$ for every effort level. Hence, e^B increases in $E(s)$. On the other hand, the change in the right-hand side of Equation (1.28) is undetermined since

$$\frac{\partial RHS}{\partial E(s)} = 1 - r_0 - \frac{r'^2[E(s)+\frac{\Delta s}{2}-z]^2}{2\gamma_2\Delta s} + \frac{r'\gamma_1(\bar{t}-t)}{\gamma_2} \gtreqless 0$$

This is so because two opposite forces work here: effort increases in $E(s)$ (minimum price should decrease), and the net benefit of commitment increases (minimum price should increase). So we cannot say much more unless we put some additional restrictions on parameters.

From condition (1.10) we can show that $z = [E(s) + \Delta s/2](1 - r(e))/(2 - r(e))$. Hence, $[E(s) + \Delta s/2 - z] = [E(s) + \Delta s/2]/(2 - r(e))$. Taking into account that effort increases in $E(s)$, this term also increases in $E(s)$. Therefore β^B increases in $E(s)$.

The left-hand side of Equation (1.27) decreases in Δs for every effort level because

$$\frac{\partial LHS}{\partial \Delta s} = \frac{-r'(E(s) + \frac{\Delta s}{2})(E(s) - \frac{\Delta s}{2})}{2(2 - r_0 - r'e)^2 \Delta s^2} = \frac{-r'\bar{s}\underline{s}}{2(2 - r_0 - r'e)^2 \Delta s^2} < 0$$

Hence, e^B decreases, and $(1 - r(e))/(2 - r(e))$ increases, in Δs. This last effect together with the initial increase in Δs implies that the minimum price increases.

The second term of the equation for β^B (1.26) is inversely proportional to the left-hand side of equation (1.27), so it increases in Δs. Moreover, effort decreases in Δs, which causes β^B to decrease in Δs. *Q.E.D.*

Figure 1.1: First-Order Conditions for Bargaining Effort (Equation (1.27))

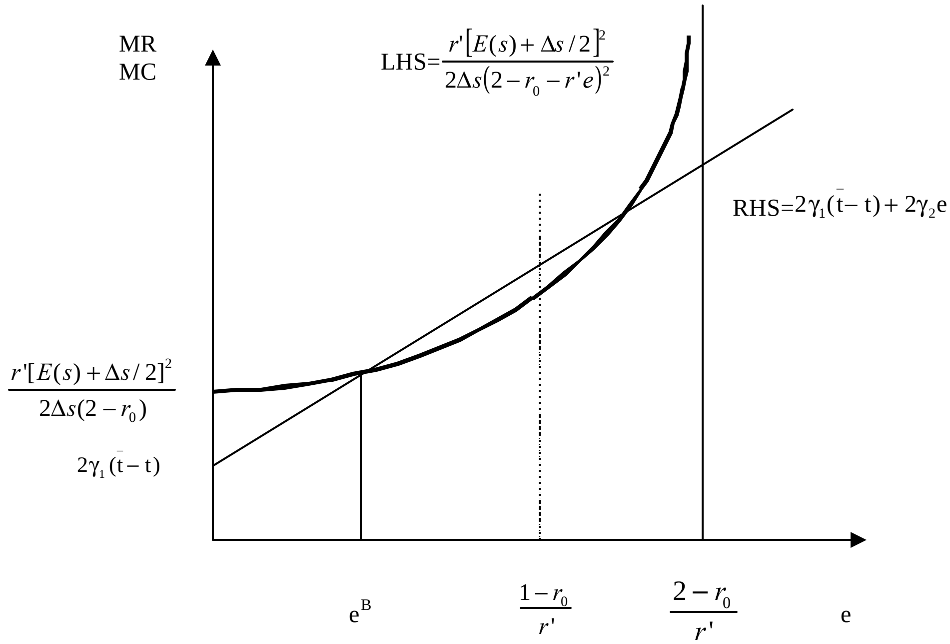
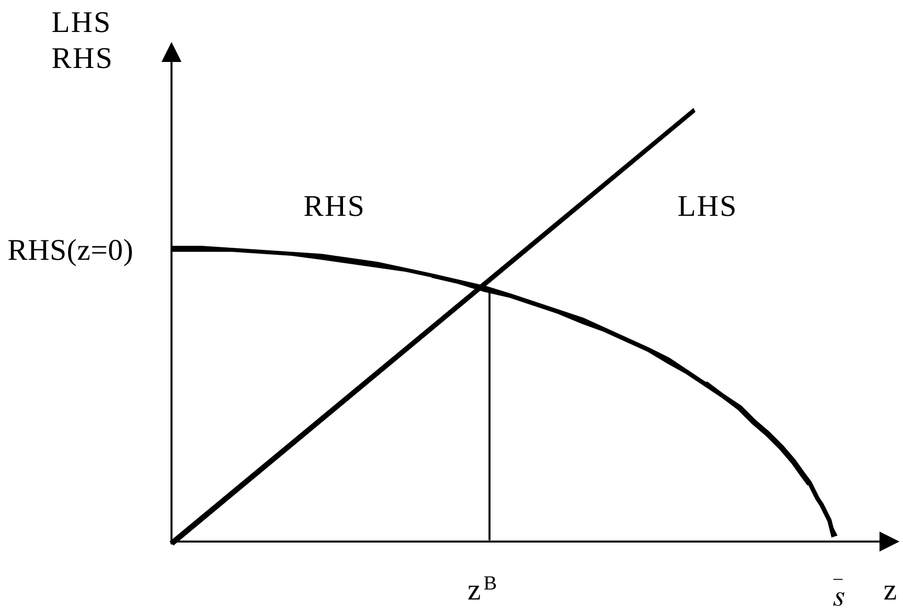


Figure 1.2: First-Order Conditions for Minimum Price (Bargaining Effort Case)



The equations come from (1.28).

$$RHS = \left\{1 - r_0 - \frac{r'^2 \left(E(s) + \Delta s/2 - z\right)^2}{4\gamma_2 \Delta s} + \frac{r'\gamma_1 \left(\bar{t} - t\right)}{\gamma_2}\right\} \left(E(s) + \Delta s/2 - z\right)$$

$$RHS(z = 0) = \frac{\left[(1 - r_0)\,\gamma_2 + r'\gamma_1 \left(\bar{t} - t\right)\right] \left(E(s) + \Delta s/2\right)}{\gamma_2} - \frac{r'^2 \left(E(s) + \Delta s/2\right)^3}{4\gamma_2 \Delta s}$$

Table 1.4: A Numerical Example of Bargaining Effort (No Commitment, $z = 0$)

t	e*	r(e*)	b(e*)	EU_P	EU_D	C(e*)	EU_B
0	1.33	0.43	0.833	176.0	0.0	32.0	272.0
0.1	1.40	0.44	0.850	175.4	2.2	33.6	268.8
0.2	1.47	0.45	0.867	174.5	4.7	35.2	265.6
0.3	1.53	0.45	0.883	173.4	7.4	36.8	262.4
0.4	1.60	0.46	0.900	172.2	10.2	38.4	259.2
0.5	1.67	0.47	0.917	170.7	13.3	40.0	256.0
0.6	1.73	0.47	0.933	169.0	16.6	41.6	252.8
0.7	1.80	0.48	0.950	167.0	20.2	43.2	249.6
0.8	1.87	0.49	0.967	164.9	23.9	44.8	246.4
0.9	1.93	0.49	0.983	162.6	27.8	46.4	243.2
1	2.00	0.50	1.000	160.0	32.0	48.0	240.0
Ex-ante Value	**1.67**	**0.47**	**0.917**	**169.6**	**14.4**	**40.0**	**256.0**

Basic Information: $\bar{s} = 950$, $\underline{s} = 10$, $E(s) = 480$, $\Delta s = 940$, $\gamma_1 = 8$, $\gamma_2 = 12$, $r' = 0.1$, $r_0 = 0.3$, $U_D(\underline{t}) = 0$.
Revenue Function: $r(e) = r_0 + r'e$, Cost Function: $C(e, t) = \gamma_1(1 - t)e + \gamma_2 e^2$.
Notation: t: type; e^*: effort; $r(e^*)$: bargaining share; $\beta(e^*)$: commission; EU_P: seller's utility; EU_D: delegate's utility; $C(e^*)$: effort cost; EU_B: buyer's utility.

Table 1.5: A Numerical Example of Bargaining Effort (Commitment Effect)

t	e^B	$r(e^B)$	z^B	b^B	EU_P	EU_D	$C(e^B)$	EU_B	Surplus (*)	Welfare Loss (**)
0	0.03	0.303	390.3	0.520	282.6	0.0	0.2	116.2	399.0	81.0
0.1	0.10	0.310	387.9	0.572	283.1	0.2	0.8	116.0	400.0	80.0
0.2	0.17	0.317	385.4	0.623	283.3	0.6	1.5	115.7	401.0	79.0
0.3	0.25	0.325	383.0	0.673	283.2	1.2	2.1	115.5	402.0	78.0
0.4	0.32	0.332	380.5	0.722	283.0	2.0	2.8	115.3	403.0	77.0
0.5	0.39	0.339	378.0	0.770	282.5	3.1	3.4	115.0	404.1	75.9
0.6	0.46	0.346	375.5	0.818	281.8	4.5	4.1	114.7	405.1	74.9
0.7	0.54	0.354	372.9	0.865	280.8	6.0	4.8	114.5	406.1	73.9
0.8	0.61	0.361	370.3	0.910	279.6	7.8	5.5	114.2	407.1	72.9
0.9	0.68	0.368	367.7	0.956	278.2	9.9	6.2	113.9	408.1	71.9
1	0.76	0.376	365.1	1.000	276.5	12.1	6.9	113.6	409.2	70.8
Ex-ante Value	**0.39**	**0.339**	**377.9**	**0.766**	**281.3**	**4.3**	**3.5**	**115.0**	**404.1**	**75.9**

Probability of bargaining failure	**39.1%**
Increase in Seller's Utility with respect to no commitment	**65.9%**
Welfare Loss/Expected Surplus	**15.8%**

Basic Information: $\bar{s} = 950$, $\underline{s} = 10$, $E(s) = 480$, $\Delta s = 940$, $\gamma_1 = 8$, $\gamma_2 = 12$, $r' = 0.1$, $r_0 = 0.3$, $U_D(\underline{t}) = 0$.
Revenue Function: $r(e) = r_0 + r'e$, Cost Function: $C(e,t) = \gamma_1(1-t)e + \gamma_2 e^2$.
Notation: t: type; e^B: effort; $r(e^B)$: bargaining share; z^B: minimum price; $\beta(e^B)$: commission; EU_P: seller's utility; EU_D: delegate's utility; $C(e^B)$: effort cost; EU_B: buyer's utility.
$(*)$ Surplus is the sum of the delegate's cost and the seller, delegate and buyer's utility.
$(**)$ Welfare loss is equal to Expected Surplus (480) minus Surplus. It can also be computed as the expected surplus between s and z.

Table 1.6: A Numerical Example of Marketing Effort (No Commitment, $z = 0$)

t	e*	p(e*)	b(e*)	EU_P	EU_D	C(e*)	EU_B	Surplus (+)
0	0.33	0.333	0.667	76.0	0.0	4.0	80.0	160.0
0.1	0.40	0.340	0.700	76.2	0.6	4.8	81.6	163.2
0.2	0.47	0.347	0.733	76.1	1.5	5.6	83.2	166.4
0.3	0.53	0.353	0.767	75.8	2.6	6.4	84.8	169.6
0.4	0.60	0.360	0.800	75.4	3.8	7.2	86.4	172.8
0.5	0.67	0.367	0.833	74.7	5.3	8.0	88.0	176.0
0.6	0.73	0.373	0.867	73.8	7.0	8.8	89.6	179.2
0.7	0.80	0.380	0.900	72.6	9.0	9.6	91.2	182.4
0.8	0.87	0.387	0.933	71.3	11.1	10.4	92.8	185.6
0.9	0.93	0.393	0.967	69.8	13.4	11.2	94.4	188.8
1	1.00	0.400	1.000	68.0	16.0	12.0	96.0	192.0
Ex-ante Value	**0.67**	**0.367**	**0.833**	**73.6**	**6.4**	**8.0**	**88.0**	**176.0**

Basic Information: $\bar{s} = 950$, $\underline{s} = 10$, $E(s) = 480$, $\Delta s = 940$, $\gamma_1 = 8$, $\gamma_2 = 12$, $p' = 0.1$, $p_0 = 0.3$, $r_0 = 0.5$, $U_D(\underline{t}) = 0$.
Probability Function: $p(e) = p_0 + p'e$, Cost Function: $C(e, t) = \gamma_1(1 - t)e + \gamma_2 e^2$.
Columns 2 and 4 are computed as those in Table 1.4, but replacing $r_0 p'$ for r'.
Notation: t: type; e^*: effort; $r(e^*)$: bargaining share; $\beta(e^*)$: commission; EU_P: seller's utility; EU_D: delegate's utility; $C(e^*)$: effort cost; EU_B: buyer's utility.
(+) Surplus is the sum of the delegate's cost and the seller, delegate and buyer's utility. The ex ante revenue in Expected Utilities and Surplus is the corresponding revenue times the probability of the delegate finding a buyer.

Table 1.7: A Numerical Example of Marketing Effort (Commitment Effect)

t	e^M	$p(e^M)$	z^M	b^M	EU_P	EU_D	$C(e^M)$	EU_B	Surplus (*)	$p(e^M)E(s)$	Loss from failure(**)	Welfare Loss v. No commitment (***)
0	0.67	0.367	316.7	2.250	106.7	0.0	10.7	39.1	156.5	176.0	19.5	3.5
0.1	0.73	0.373	316.7	2.325	106.6	1.2	11.7	39.8	159.3	179.2	19.9	3.9
0.2	0.80	0.380	316.7	2.400	106.3	2.6	12.8	40.5	162.2	182.4	20.2	4.2
0.3	0.87	0.387	316.7	2.475	105.7	4.2	13.9	41.3	165.0	185.6	20.6	4.6
0.4	0.93	0.393	316.7	2.550	105.0	6.0	14.9	42.0	167.8	188.8	21.0	5.0
0.5	1.00	0.400	316.7	2.625	104.0	8.0	16.0	42.7	170.7	192.0	21.3	5.3
0.6	1.07	0.407	316.7	2.700	102.8	10.2	17.1	43.4	173.5	195.2	21.7	5.7
0.7	1.13	0.413	316.7	2.775	101.5	12.7	18.1	44.1	176.4	198.4	22.0	6.0
0.8	1.20	0.420	316.7	2.850	99.9	15.4	19.2	44.8	179.2	201.6	22.4	6.4
0.9	1.27	0.427	316.7	2.925	98.0	18.2	20.3	45.5	182.1	204.8	22.7	6.7
1	1.33	0.433	316.7	3.000	96.0	21.3	21.3	46.2	184.9	208.0	23.1	7.1
Ex-ante Value	**1.00**	**0.400**	**316.7**	**2.625**	**102.9**	**9.1**	**16.0**	**42.7**	**170.7**	**192.0**	**21.3**	**5.3**

Probability of bargaining failure	**32.6%**
Loss from bargaining failure (**)	**11.1%**
Increase in Seller's Utility with respect to no commitment	**39.9%**
Welfare Loss/Expected Surplus	**3.0%**

Basic Information: $\bar{s} = 950$, $\underline{s} = 10$, $E(s) = 480$, $\Delta s = 940$, $\gamma_1 = 8$, $\gamma_2 = 12$, $p' = 0.1$, $p_0 = 0.3$, $r_0 = 0.5$, $U_D(\underline{t}) = 0$.
Probability Function: $p(e) = p_0 + p'e$, Cost Function: $C(e,t) = \gamma_1(1-t)e + \gamma_2 e^2$.
Notation: t: type; e^M: effort; $p(e^M)$: probability of finding a buyer; z^M: minimum price; $\beta(e^M)$: commission; EU_P: seller's utility; EU_D: delegate's utility; $C(e^M)$: effort cost; EU_B: buyer's utility. $(*)$ Surplus is the sum of the delegate's cost and the seller, delegate and buyer's utility. $(**)$ Loss from bargaining failure is equal to Expected Surplus $(pE(s))$ minus Surplus. It can also be computed as the expected surplus between s and z.
$(***)$ Welfare Loss against No Commitment is the difference between Surplus under No Commitment and Surplus under Commitment.

Chapter 2

Ex Post Monitoring and Collusion

2.1 Introduction

In many economic situations involving agency relationships, monitoring has been shown to be an important instrument for the principal to alleviate incentive problems. The principal hires a third party (a monitor) to obtain a signal about the agent's private information or actions, which is used to increase output (or reduce cost) and reduce the agent's rent. However, when the monitor is self-interested and may collude with the agent, his worthiness to the principal may be reduced. In this setting, interesting questions arise regarding the principal's choice of the monitor and his function. In particular, is the timing to hire a monitor (i.e., to monitor the agent before or after the outcome is realized) relevant to the principal? Should monitoring be performed on agent's effort or productivity? If the monitor is self-interested, how the answer to the previous questions is affected when there is a threat of collusion between the agent and the monitor?

This chapter answers the second question by comparing effort monitoring and productivity auditing,[1] and briefly discusses the third question in the ex-post-monitoring case. The first question (and the corresponding third question) is addressed in the next chapter.

We show that all parties (principal, agent and monitor) obtain the same utility under either effort monitoring or productivity auditing, provided that some conditions on the distribution of the monitor's signal, the monitoring cost and efficiency of the side transfers between the monitor and the agent are satisfied, and that the monitor is hired after the outcome is realized. Second, hiring a self-interested monitor introduces potential collusion between monitor and agent. Given the previous result and that the benefits from collusion come from modifying the monitor's report, the cost to prevent collusion is the same to the principal regardless of what is being monitored (i.e., effort or productivity).

We present our results in a principal-agent framework with moral hazard and adverse selection. It is well known that the principal generates inefficiencies in order to mitigate the asymmetric information problem (see for example Baron and Myerson [3]). In addition, the principal may hire an ex post monitor to obtain information (a signal) about the agent's private characteristic or effort and therefore reduce the agent's information rent (Baron and Besanko [2] is an excellent treatment of this point). An additional problem arises when the supervisor can collude with the agent. When the agent is harmed by the monitor's signal, she may have incentives to bribe

[1] This comparison is meaningful since both monitoring activities occur after the agent exerted effort. There is no point in addressing this question before the agent exerts effort.

the latter. The threat of collusion depends strongly on the timing of monitoring, the degree of enforceability of side-contracts and the characteristics of the monitor's information. Kessler [33] shows that collusion can be avoided at no additional cost when the monitor's information is hard (i.e., observable and verifiable) and non-forgeable in an effort-monitoring setting. Kofman and Lawarrée [38] show that when information can be forged, the collusion constraint increases the costs faced by the principal when designing a contract for the other parties (this is done in a productivity-auditing setting).[2] From our previous result, we conclude that it is the ex post nature of the monitoring relationship (and not what is being monitored) what drives these results.

The rest of the chapter is as follows. Section 2.2 introduces the model and the monitoring technology. Section 2.3 presents the main result of principal's indifference between the two monitoring technologies. Section 2.4 concludes.

2.2 Model

Consider a three-layer hierarchy: principal-monitor-agent. The principal hires the agent to produce some output x for her (for instance, assume that the hierarchy is a firm and x is profit). The agent exerts effort e and has private productivity θ ($\theta \in \{\theta_L, \theta_H\}, \theta_H > \theta_L > 0$). A type-$\theta_H$ (type-θ_L) agent is more (less) productive. Both productivity and effort are not observed by the principal. The agent's cost or

[2] When the monitor supervises the agent before effort is exerted, Tirole [56, 58] shows that collusion costs cannot be avoided.

disutility of effort is $g(e)$, increasing and strictly convex, with $g(0) = g'(0) = 0$.[3] Let q be the ex ante probability that the agent is type θ_H, i.e., $q = Pr(\theta = \theta_H)$. The agent produces an observable output or profit $x = \theta + e$, and is paid a compensation t. Her reservation utility is simplified to $\underline{U} = 0$.

After observing the outcome the principal decides whether to hire a costless monitor.[4] We consider two possibilities. First, the monitor observes an imperfect signal of the agent's type that may take one out of three different values. With probability $1 - p$, the monitor learns nothing about the agent's type ($\sigma = 0$). Otherwise he obtains a signal, which is imperfectly correlated with the agent's true type. The signal σ is correct with probability $Pr(\sigma = \theta|\theta) = \alpha > 1/2$. The principal obtains a report r from the monitor ($r \in \{0, L, H\}$, where L corresponds to θ_L and H corresponds to θ_H) and compares it with the agent's report ($\hat{\theta}$). The agent is punished when she is found having misreported her type. The agent is protected by limited liability: the punishment z^r must be at most z. The bound z may be either low (the agent is protected by some minimum legal wage), or high (wealth or legal constraints). We assume that the monitor's compensation is non-negative ($t \geq 0$).

The second possibility corresponds to effort monitoring (as in Kessler [33]). The monitor obtains a signal σ about the agent's effort. The principal sets a benchmark effort $\tilde{e} \in (e_l - \Delta\theta, e_l)$ (where $\Delta\theta = \theta_H - \theta_L > 0$) to detect shirking. As we will show

[3] An extra assumption $g'''(e) \geq 0$ guarantees that the optimal contract is deterministic.

[4] This is a simplifying assumption that does not change our main result (conditioned on the monitor being hired) when there is a positive monitoring cost.

later, if the type-θ_L agent exerts effort e_l, a type-θ_H agent will exert effort $e_l - \Delta\theta$ if she shirks. With probability $1-p$ the monitor learns nothing about the agent's effort ($\sigma = 0$). Otherwise he observes that the agent shirked ($e < \tilde{e}$) or did not shirk ($e \geq \tilde{e}$). Conditional on the effort exerted by the agent, the signal is correct with probability $\alpha > 1/2$.

The timing of the game is as follows: After nature chooses the agent's type θ, the principal offers a set of contracts $\{t(x,r), z^r\}$ to the agent and $w(x,r)$ to the monitor. The three parties sign the contract. If the agent accepts the contract, she exerts effort e to produce an output x. The principal sends the monitor with some probability δ to observe a signal σ about the agent's type. If they find it profitable, monitor and agent meet to bargain over the monitor's report (in whose case they sign a side contract for a transfer b from the agent to the monitor, who values it $v(b) = kb$, $0 \leq k \leq 1$). Then the monitor reports r to the principal. Finally, transfers t, w, punishment z and side transfers are realized.

Assume that α, p, k and the monitoring cost are the same under effort monitoring and productivity auditing to rule out principal's preference of a particular technology because of its reliability or cost.

All parties are risk-neutral. The agent, supervisor and principal's utility is $U_A = w - b - g(e)$, $U_S = w + b$ and $U_P = x - t - w$, respectively. The two benchmark cases without monitor are: i) first best contract, and ii) contract under asymmetric information with no monitor.

i) First Best: If the principal observes both agent's effort and type, she designs a contract that maximizes her utility $U_P = \theta + e - t$, subject to the agent's interim participation constraint ($t - g(e) \geq 0$). The first-best (forcing) contract is an effort recommendation e^{FB} that satisfies $g'(e^{FB}) = 1$ and a wage $w = g(e^{FB})$.

ii) No Monitor: When the principal does not observe both agent's effort and type, she has to design a contract based only on the observable x. According to the revelation principle (Baron and Myerson [3]), the principal can restrict herself to a direct mechanism. Then, for an agent's report $\hat{\theta}$ there is an effort recommendation $e(\hat{\theta})$ to produce a profit $x(\hat{\theta}) = \hat{\theta} + e(\hat{\theta})$. Let t_j the wage and e_j the effort when the agent reports $\hat{\theta} = \theta_j$, $j \in \{L, H\}$. The contract must satisfy agent's participation and incentive compatibility constraints. Let $\Delta\theta = \theta_H - \theta_L$. The incentive compatibility constraints are

$$t_L - g(e_L) \geq t_H - g(e_H + \Delta\theta) \qquad \text{and} \qquad t_H - g(e_H) \geq t_L - g(e_L - \Delta\theta)$$

The optimal contract is such that only the type-θ_L participation constraint and the type-θ_H incentive compatibility constraint are binding. In order to elicit high effort from the type-θ_H agent, the principal pays her an information rent. To reduce this rent, she elicits lower effort from the type-θ_L agent, who earns no rent.[5]

[5] Assume $e_L \geq \Delta\theta$ to make the incentive compatibility constraint meaningful.

2.3 Ex Post Monitoring

A monitor is of value to the principal since his report can be used to reduce the type-θ_H agent's wage and increase the type-θ_L agent's effort. Consider first the case of a benevolent monitor.

2.3.1 Benevolent Monitor

Suppose that the monitor audits the agent's productivity (auditor) and reports his signal honestly. The principal does not send the auditor when she observes a high output since the type-θ_L agent does not benefit from pretending to be more efficient (incentive compatibility ensures that *only* a type-θ_H agent produces high output). But the principal sends the auditor (with probability $\delta \in (0, 1]$) when she observes a low output because the type-θ_H agent *does* benefit from shirking. According to the revelation principle, the principal can restrict herself to ask for an agent's truthful report $\hat{\theta} \in \{\theta_L, \theta_H\}$ and propose an effort and compensation $\{e(\hat{\theta}), t(\hat{\theta}, x), z^r\}$. Let e_h and t_h (e_l and t_l) denote the agent's effort and compensation when the agent reports $\hat{\theta} = \theta_H$ ($\hat{\theta} = \theta_L$), and let z^r be the punishment to the agent when the auditor reports $r \in \{0, H\}$.[6] Note that $x_l = \theta_L + e_l$. This outcome occurs when the type-θ_L agent reports truthfully or when the type-θ_H agent shirks since she produces x_l to pretend to be less productive. Also, note that $x_h = \theta_H + e_h$. The agent's constraints are

[6] Given that principal sends the monitor to audit a type-θ_L agent in equilibrium, she does not punish the agent when monitor's and agent's reports coincide.

$$
\begin{aligned}
&\text{IR(L)}: && t_l - \delta\left[p(1-\alpha)z^H + (1-p)z^0\right] - g(e_l) \geq 0 \\
&\text{IR(H)}: && t_h - g(e_h) \geq 0 \\
&\text{IC(L)}: && t_l - \delta\left[p(1-\alpha)z^H + (1-p)z^0\right] - g(e_l) \geq t_h - g(e_h + \Delta\theta) \\
&\text{IC(H)}: && t_h - g(e_h) \geq t_l - \delta\left[p\alpha z^H + (1-p)z^0\right] - g(e_l - \Delta\theta) \\
&\text{LL}: && z^r \leq z, \qquad r \in \{0, H\}
\end{aligned} \tag{2.1}
$$

Since the auditor is honest, the principal obtains the information at no cost, i.e., $w^r = 0$, for $r \in \{0, L, H\}$. Let - $= \{e_h, e_l, t_h, t_l, z^H, z^0, \delta\}$ be the set of choice variables. The principal's problem is

$$
\max EU_P = q(\theta_H + e_h - t_h) + (1-q)\Big\{\theta_L + e_l - t_l + \delta\left[p(1-\alpha)z^H + (1-p)z^0\right]\Big\} \tag{2.2}
$$

subject to constraints (2.1). The optimal contract is

$$
\begin{aligned}
&g'(e_h) = 1 && e_h = e^{FB} \\
&g'(e_l) = 1 - \tfrac{\lambda_3}{1-q}\left\{g'(e_l) - g'(e_l - \Delta\theta)\right\} && e_L \leq e_l \leq e^{FB} \\
&t_l = g(e_l) + (1 - p\alpha)z \\
&t_h = \begin{cases} g(e_h) + g(e_l) - g(e_l - \Delta\theta) - p(2\alpha - 1)z & \text{if} \quad \lambda_3 = q \\ g(e_h) & \text{if} \quad \lambda_3 < q \end{cases} \\
&z^H = z^0 = z \qquad w^r = 0, \quad r \in \{0, H\} && \delta = 1
\end{aligned} \tag{2.3}
$$

where $\lambda_3 \in [0, q]$ is the shadow price of the IC(H) constraint. The proof of this result is similar to that of Proposition 10 in next chapter, and hence is omitted.

Now consider the agent's constraints under effort monitoring (described in Section 2.2). In order to compare the principal's problem under effort monitoring with that under productivity auditing, let z^H denote the punishment to the agent when the monitor reports that the agent shirked (i.e., when the monitor observed $e < \tilde{e}$), and z^0 the punishment to the agent when the monitor reports that he found nothing. The agent's participation, incentive compatibility and limited liability constraints are the same as (2.1) and hence the principal's problem is exactly (2.2). Then we have the following result:

Proposition 8 *The optimal contract when the principal hires a benevolent monitor to audit the agent's private information* <u>*after*</u> *the output is realized is the same as that when the principal hires him to monitor the agent's effort.*

This proposition highlights that what is relevant to the principal is ex post monitoring rather than its particular characteristics, provided that the signals from effort monitoring and productivity auditing are equally informative and costly.[7]

2.3.2 Self-Interested Monitor

Problem (2.2) in the previous section is the building block to analyze the optimal contract under collusion when the principal hires a monitor after the agent exerted effort. We showed that productivity auditing and effort monitoring are utility-equivalent to all the players if the accuracy of the monitor's signal and the cost to obtain it are the same for both monitoring technologies.

[7] In next chapter we use this result, together with those in next section, to simplify the monitoring timing discussion to ex ante and ex post monitoring.

When the monitor is self-interested, the threat of collusion between the monitor and the agent becomes relevant to the principal. Both agent and monitor will bargain over the monitor's report and sign a side-contract whenever they find it profitable to do so. Hence, additional constraints are added to (2.1).

If the monitor's signal, which is also observed by the agent, is "hard and non-forgeable" (as in Tirole [56]), the monitor can conceal his information, but cannot forge it either with help from the agent or by blackmailing her. The principal may want to prevent the agent-monitor coalition from changing a report $r \in \{L, H\}$ to $r = 0$.[8] On the other hand, if information is hard and forgeable (as in Kofman and Lawarrée [38]) or (as in Baliga [1]) the coalition may manipulate the monitor's report, and the principal may want to prevent changes from $r \in \{0, H\}$ to $r = L$ (in whose case the agent is not punished). In all these cases, the collusion constraints involve avoiding changes in the monitor's report.[9] Then, we extend the result from the previous section to the case of collusion: *Ex post monitoring does not depend on what is being observed by the monitor (i.e., agent's productivity or effort) when the monitoring technology, efficiency of side transfers and monitoring cost are the same.*

Consider now that the monitoring technologies are different. Let $\{p_P, \alpha_P, k_P, c_P\}$ denote the probability, informativeness of the signal about the agent's productivity,

[8] This measure is not necessary in some cases. For example, the coalition will not find profitable to hide a report $r = L$, since the agent is not punished in that case, and may be punished otherwise.

[9] In this section we only show that the agent-monitor coalition has incentives to modify the monitor's report. In next chapter we discuss how this problem is solved by the principal (see Sections 3.4 to 3.6.

side-transfer efficiency parameter and cost to obtain the signal, and let $\{p_E, \alpha_E, k_E, c_E\}$ denote those corresponding to a signal about the agent's effort.

If a particular technology (e.g., productivity auditing) generates a more informative signal (i.e., $\alpha_P > \alpha_E$ and/or $p_P > p_E$), or is less expensive to obtain (i.e., $c_P < c_E$ or $k_P < k_E$), it will dominate the other monitoring technology. Note that this result does not depend on the linear technology chosen in this chapter (see Chapter 3).

2.4 Conclusion

In this chapter we show that the optimal contract with and ex post monitor does not depend on what variable is being monitored (agent's effort or productivity), provided that the signals of effort and type are equally accurate and costly.

This result extends to the case of hiring a self-interested monitor, independently of the characteristics of the monitor's information (hard and non-forgeable, hard and forgeable or soft), because the threat of collusion between the monitor and the agent corresponds to changes in the monitor's report (which does not depend on the variable observed).

The next chapter analyzes the trade off that arises from the monitoring timing choice (before or after the agent exerted effort) under different collusion environments, which are defined by the characteristics of the monitor's information.

Chapter 3

Optimal Monitoring Timing under Collusion

3.1 Introduction

The literature on the principal-agent problem has analyzed the role that monitoring institutions (usually being exposed to collusion with agents) play in alleviating incentive problems. Two branches of this literature have been studied separately. The first one analyzes the effects of hiring a supervisor on the agent's incentives and on the principal's contract design (see, for example, Tirole [56, 58] and Laffont and Tirole [43], ch. 11). The supervisor obtains information about the agent's productivity *before* the agent exerts effort (we also refer to him as *ex ante* monitor). The second branch of the literature studies the optimal contract when the principal hires an auditor (see, for example, Baron and Besanko [2], Kofman and Lawarrée [38] and Laffont and Tirole [43], ch. 12). The auditor obtains information about the agent's effort or productivity *after* the agent exerted her effort (we also refer to him as *ex post* monitor). We will refer to both of them as monitors when there is no need to distinguish them.[1]

However, not much attention has been given to what affects the choice between the two monitoring institutions.[2] There are reasons that justify the importance of this problem. First, the evidence suggests the existence of both institutions. Owners of firms hire third parties to supervise employees, monitor their effort or audit their private information. In regulation, governmental agencies intervene in industries (by means of ex ante controls) or audit firms' accounting data and expenses. Second, *the monitoring timing choice under collusion is not inconsequential since it has different effects on the agent's incentives, the stakes in collusion between the agent and the monitor, and therefore the principal's utility*. On the one hand, the supervisor's report is used to design a "flexible" contract for the agent, in which output and the agent's compensation are based on both the agent's and monitor's report. This creates incentives for the agent to bribe the supervisor when the agent's rents are reduced, which restricts the principal in the contract design. On the other hand, the principal can use the auditor's report to punish the agent (provided that a punishment scheme is available), but she cannot make the contracting of output depend on this report. In this case, the agent has incentive to bribe the auditor to avoid being punished. The principal faces a trade-off between flexibility and rigidity-punishment, and the threat of collusion is an important determinant of her decision.

[1] Throughout the chapter we discuss the case of hiring one monitor and concentrate on monitoring timing. The case of joint monitoring is left to future research.

[2] An exception is Shavell [53], Kolstad *et al.* [39] and the references therein, who analyze the stage of legal intervention or (benevolent) regulation of activities that generate externalities.

We find the optimal solution to this trade-off under different informational environments. Special importance is given to the manipulation of the monitor's information, which determines the quality of such information and the cost for the principal to obtain a truthful report from the monitor.

In particular, we specify a general model (principal-monitor-agent hierarchy) that allows for both supervising and auditing. The principal, who is uninformed about the agent's productivity and effort, hires the agent to produce a good or service. In addition, the principal may hire a self-interested monitor to extract some of the agent's private information, which can be used to reduce the agent's rents. As a response, the agent may bribe the monitor to change his report. This creates additional constraints to the principal depending on the structure of the monitor's signal (which determines the quality of the information and the cost to the principal to obtain it) and the degree of enforceability of the side-contract. Using the classification from the literature, we consider three different informational environments:

(i) "hard and non-forgeable" information, which means that the monitor has verifiable proof of his signal and may conceal it from the principal (see Tirole [56, 58])

(ii) "hard and forgeable" information, which means that the monitor can falsify his signal with help from the agent (see Kofman and Lawarrée [38])

(iii) "soft" information, which means that the monitor has no verifiable proof of his signal, and hence may report anything (see Baliga [1], Faure-Grimaud *et al.* [15]).

The third case is a more serious problem to the principal than the second case is (which is more serious than the first). The quality of the monitor's information is lower when it becomes "less reliable". Consequently, the cost to the principal of obtaining a truthful report from the monitor is higher. Nevertheless, it is important to study each of these cases individually. Depending on the activity to be controlled, the monitor may "ignore" relevant information to write his report (such as not reporting perquisites), or may not obtain information when it is hard to dispose of.[3] These are examples of hard and non-forgeable information. In addition, the monitor may write reports based on evidence pre-selected by the manager (such as audit reports in a company's credit department), or distort information (alter payrolls, create fictitious personnel, manipulate quality tests, etc., see Dalton [12], p. 32). These last two cases correspond to hard and forgeable information and soft information, respectively.

A general result that emerges from the optimal solution to the principal's trade-off is that the principal more probably hires the monitor to supervise the agent when the quality of the monitor's information (measured by the degree of manipulation of it) is poor and side transfers between the monitor and the agent are costly to be enforced. As the quality of information is better or side transfers are more efficient, the auditor

[3] Dalton [12] described that "...safety and health inspectors usually telephoned in advance of visits so that they would not see unsafe practices or conditions they would feel obliged to report." (p. 48).

becomes more valuable to the principal.

In particular, the solution to this trade-off when the monitor and the agent cannot enforce side-contracts is such that auditing is optimal when the principal has strong punishment schemes or when the punishment is weak and the monitor's signal is noisy. Otherwise, supervising is optimal when the punishment instrument is weak and the signal about the agent's type is very informative. Given low punishment, the supervisor is more valuable to the principal when he learns the "right" information about the agent (which is more probable when his signal is accurate), for the principal can reduce the agent's rents when she is certain about the agent's type. The principal's choice is summarized in Table 3.1.

First, with hard information and enforceable collusive contracts, the threat of collusion imposes no additional cost to auditing but does increase the cost of supervising. So collusion with hard information makes auditing more likely to dominate supervising. Consider the contract under no collusion. The agent has an incentive to bribe the supervisor to hide his signal when the agent's rents are reduced (which happens when the supervisor observes the agent's type correctly). The principal has to incur in higher costs to obtain a truthful report. With an auditor, the principal designs a contract such that the agent is punished whenever the auditor does not find favorable information (so the agent is also punished when the auditor observes nothing). Incentives to conceal the auditor's information are eliminated at no additional cost. Nevertheless, supervising is still optimal for weak penalties and precise signal.

Table 3.1: Monitoring Timing and Collusion: Summary of Results

No Collusion/Collusion with Hard and Non-Forgeable Information		
	Weak Punishment	Strong Punishment
Noisy Signal		
Efficient Side Transfer*	Auditor	Auditor
Inefficient Side Transfer*	Auditor	Auditor
Informative Signal		
Efficient Side Transfer*	Auditor	Auditor
Inefficient Side Transfer*	Supervisor	Auditor

Collusion with Hard and Forgeable Information or Soft Information		
	Weak Punishment	Strong Punishment
Noisy Signal		
Efficient Side Transfer	Auditor	Auditor
Inefficient Side Transfer	Supervisor	Supervisor
Informative Signal		
Efficient Side Transfer	Auditor	Auditor
Inefficient Side Transfer	Supervisor	Auditor

$*$ The efficiency of side transfers classification corresponds to the Collusion with Hard and Non-Forgeable Information case. The No Collusion case corresponds to the limiting case of inefficient (or non-enforceable) side transfers.

Second, when information can be forged, collusion imposes additional costs to both supervising and auditing. The effect of collusion on the principal's timing choice is ambivalent. On the one hand, the cost to the principal of obtaining a truthful supervisor's report is higher than that under non-forgeable information. On the other hand, this cost becomes positive when the principal hires an auditor.[4] Supervising

[4] The agent-auditor coalition does not benefit from concealing the auditor's signal but does

is more likely to be optimal when punishment schemes are weak (the deterrent effect of punishment is poor) for any precision of the signal, and when the signal is noisy for any level of punishment (this is the case when the principal does not benefit from an auditor's noisy signal), provided that side transfers between the agent and the monitor are inefficient.

Finally, the principal's utility and the monitoring timing under soft information are the same as those under hard and forgeable information, and hence the results in that case also hold under soft information. With soft information, the coalition parties will change their report individually or jointly when they find it profitable to do so. Joint deviations are taken care of in the forgeable information case. Moreover, the principal can design a contract that reduces the coalition rents (or punishes the coalition) when the agent and monitor's report of the monitor's signal do not coincide.

We can use these results to explain why top-level managers in organizations (such as CEOs, who may be more exposed to punishments) are usually audited, while low-level employees (typically with low incomes or protected by minimum wage regulations) are supervised during the production stage. We also apply our results to regulation of "hazardous" activities, and discuss the optimality of the ex ante stage of intervention (in Law enforcement) when the regulatory agency is able to manipulate its information.

We connect many works dedicated to monitoring and/or collusion in hierarchies,

benefit from the auditor reporting the correct agent's type.

which apply to either supervising or auditing. Baron and Besanko [2] analyzed the optimal design of a regulatory contract when the government hires a benevolent regulator to audit a firm. They obtain a separation result that the pricing decision does not depend on the auditing decision (which means that the price and quantity when the firm is audited are the same as those when the firm is not audited), but the auditing decision depends on the pricing decision (in particular, the principal sends the auditor when she infers that the firm overstated the price). Tirole [56, 58] and Laffont and Tirole [42], [43] (ch. 11) analyze the optimal contract under collusion between a self-interested supervisor/regulator (or power groups) and an agent in a hard-and-non-forgeable-information framework. Kessler [33], Khalil and Lawarrée [34] and Laffont and Tirole [43] (ch. 12) analyze the effects of collusion on contract design in an auditor-based hierarchy under different information environments. Faure-Grimaud *et al.* [16] show the equivalence of two hierarchical organizations under soft information: a collusive supervisor and the delegation to the supervisor of contracting with the agent. A similar result is obtained by Faure-Grimaud *et al.* [15] in a model with an auditor. Kofman and Lawarrée [38] show that a second auditor is valuable to the principal, for he can be used to discipline an internal auditor when information is hard and forgeable. Baliga [1] shows the equivalence of the principal's utility when information is hard or soft in Tirole's [58] stylized model. (We generalize this result to an equivalence between the principal's utility under hard and forgeable information and that under soft information.) However, none of them studies the optimality of supervising as compared with auditing. Finally, as we mentioned before, the Law and Economics literature (Cohen [9], Kolstad *et al.* [39], Shavell [53] and the references therein) has analyzed the optimal stage of regulation of activities that generate externalities in a benevolent-regulator framework. We show that hiring a self-interested regulator has effects on the optimal regulatory stage.

The chapter is organized as follows. Section 3.2 outlines the model. Section 3.3 sets the benchmarks (No Monitor and No Collusion). We study the effect of collusion on the monitoring timing decision under various information structures (hard and non-forgeable, hard and forgeable and soft information) in Sections 3.4 to 3.6. Section 3.7 provides applications to organization design and regulation. Finally, Section 3.8 concludes.

3.2 Model

Consider a hierarchy consisting of an owner (principal), a monitor (supervisor or auditor) and a manager (agent).[5] The principal hires the agent to produce a good with gross value V and production cost $C = \bar{\theta} - \theta e$. The payoff to the principal is $V - \bar{\theta} + \theta e$.[6] The cost C, which is *observed* by the principal, is reduced by a combination of agent's productivity and effort, which are *not observed* by the principal. By exerting higher

[5] We also consider other hierarchies, such as owner-headman-worker, government-regulator-firm/contractor.

[6] This specification of the model nests regulation models (with cost function C) and organization models (with profit function $\pi \approx \theta e$).

effort the agent reduces the production cost, but she derives a private effort disutility or cost $\psi(e) = e^2/2$.[7] The agent's private productivity or type is $\theta \in \{\theta_L, \theta_H\}$, with $\theta_H > \theta_L > 0$. Let q be the ex ante probability that the agent's productivity is high, i.e., $q = Pr(\theta = \theta_H)$. The parameter $\bar{\theta}$ is an upper bound on the production cost.[8] The principal reimburses the cost C and pays a net transfer t to the agent. The agent's reservation utility is normalized to $\underline{U} = 0$.

The principal also decides whether to hire a monitor who observes an imperfect signal about the agent's productivity (this signal is also observed by the agent). The monitor obtains the signal at no cost (the results extend to a costly monitor, provided that he is hired). The signal may take the following values: With probability $1 - p$ the monitor learns nothing about the agent's type ($\sigma = 0$). Otherwise he gets an imperfect observation of the agent's type ($\sigma \in \{L, H\}$), which is correct with probability $\alpha > 1/2$. This assumption satisfies the monotone likelihood ratio property that a correct signal is more probable. Table 3.2 summarizes the possible signals and their corresponding probabilities.

With the new information, the principal may set a fine or reduce the agent's wage whenever she finds that the agent misreported her type or shirked. The agent is protected by limited liability when punished: an eventual fine z^r set by the principal (depending on the monitor's report r) must be up to some liability maximum z.

[7] We assume a quadratic effort cost to obtain simple solutions to the optimal contract. The results can be generalized to more general (convex) functions.

[8] We show later that the effort exerted by a type-θ_H agent is $e = \theta_H$, and hence we assume that $\bar{\theta} > \theta_H^2$ for the observed cost to be positive in all the cases analyzed here.

Table 3.2: Monitor's Signal of Agent's Type

σ	Probability	Observation	θ
0	$1-p$	0	θ_H
H	$p\alpha$	θ_H	θ_H
L	$p(1-\alpha)$	θ_L	θ_H
0	$1-p$	0	θ_L
H	$p(1-\alpha)$	θ_H	θ_L
L	$p\alpha$	θ_L	θ_L

The liability bound may be interpreted as exogenous wealth constraints or exogenous maximum legal punishment.

When her rents are reduced, the agent may bribe the monitor to change his report. The principal will be affected by this threat of collusion differently, depending on the structure of the monitor's information and the enforceability of side contracts.[9] We consider three cases of information structure. First, the signal may be "hard", in the sense that the monitor's observation is verifiable to the principal. The monitor can conceal information, but cannot forge it either with help from the agent or by blackmailing her. Second, information may be hard and forgeable. The monitor may make up a report with the agent's help. Finally, information may be soft, in whose case the monitor is not able to provide the principal with a verifiable proof of his

[9] In order to focus only on the effects of collusion on the contract design and monitoring timing, we assume that side contracts are enforceable (transfers may be inefficient) and non-renegotiable, and hence we obtain an upper (lower) bound on the utility that the coalition (principal) can achieve. For a discussion on this point, see Tirole [58].

report. We allow for *inefficient* side transfers b from the agent to the monitor (whose valuation of the transfer is $v(b) = kb$, $0 \leq k \leq 1$).

The monitor sends a report $r \in \{0, L, H\}$ to the principal, who pays him a wage w. He is protected by limited liability ($w \geq 0$). His reservation utility is normalized to 0.

In addition, the principal has to decide whether to send the monitor *before* or *after* the agent exerted effort. In the first case, the monitor supervises the agent and obtains a signal about the agent's productivity (effort has not been exerted yet). In the second case, the monitor may either audit the agent's productivity or monitor effort. Given the cost structure ($C = \bar{\theta} - \theta e$), the information obtained by the principal is the same whether monitoring generates a signal σ on productivity or effort, provided that the signals are equally precise and costly (see Chapter 2). Whether monitoring is ex ante or ex post (on effort or productivity), we assume the same distribution of signal (same p and α) and the same cost ($c = 0$) to eliminate a possible source of timing preference. Hence we concentrate on (ex ante or ex post) productivity monitoring for convenience in the exposition.

In order to make the timing decision, the principal compares costs and benefits under each alternative. If she hires a supervisor, she obtains a report that can be used to contract output and wage. This gives some flexibility to the output choice, for the principal can create distortions according to the probability of the events in order to reduce the agent's rent. On the other hand, when the principal hires an auditor, she does not benefit from the flexibility in contracting output, but she can punish the agent when she finds that the agent misreported her type (or shirked). There is a trade-off: flexibility in contracting vs. rigidity plus punishment.[10]

The timing of the game is as follows.

1. Nature chooses the agent's type θ. The agent learns her type.

2. The principal decides the monitoring timing. If a supervisor is hired, he observes a signal σ (which is also observed by the agent).

3. The principal offers a set of contracts: $t(C, r)$ to the agent ($\{t(C, r), z^r\}$ if the agent is audited) and $w(C, r)$ to the monitor. The three parties sign the contract.

4. If a supervisor is hired, he meets the agent to negotiate over his report r. Then he reports to the principal.

5. The agent chooses effort e. The cost C is realized.

6. If an auditor is hired, he (and the agent) observes a signal σ, and meets the agent to negotiate over his report r. Then he reports to the principal.

7. Transfers, punishment and side transfers are realized.

We assume that the three parties are risk neutral. Since there are both moral hazard and adverse selection, a transfer of the hierarchy from the principal to the agent is not optimal.[11] The principal, agent and monitor's utility is $U_P = V - [t + w + C]$,

[10] As we will show in detail in Sections 3.4 to 3.6, collusion imposes different cost to these decisions depending on the information structure of the monitor's signal.

[11] Limited liability to the monitor ensures that a transfer of the hierarchy from the principal to the monitor is not possible.

$U_A = t - b - e^2/2$ and $U_M = w + kb$, respectively.

When the principal observes both agent's effort and type, the problem simplifies to choose effort and transfers in order to maximize $V - \bar{\theta} + [\theta e - t]$, for $\theta \in \{\theta_L, \theta_H\}$, subject to the agent's interim participation constraint $t - e^2/2 \geq 0$. The solution to this problem is: $e_j^{FB} = \theta_j$, $t_j^{FB} = \theta_j^2/2$, for $j = L, H$. The principal's expected utility is $EU_P^{FB} = V - \bar{\theta} + \left[q\theta_H^2 + (1-q)\theta_L^2\right]/2$.

3.3 Benchmarks: No Monitor, No Collusion

In this section we present the two relevant benchmarks of the timing problem. The lower bound on the principal's utility is achieved by contracting with the agent directly. The upper bound is achieved when the agent and the monitor cannot sign enforceable side-contracts ($k = 0$), so that the principal obtains the monitor's signal at no cost.

3.3.1 No Monitor

Suppose that the principal does not observe either agent's effort or type. The contract offered by the principal should be conditioned only on the observable C. Because of the binary nature of the problem and the fact that C is deterministic for a given agent's type, we can concentrate on forcing contracts. As it is well known from revelation principle, the principal can restrict herself to Bayesian direct mechanisms based on an agent's truthful report. For a report $\hat{\theta}$ there is an effort recommendation

$e(\hat{\theta})$ to achieve a production cost $C(\hat{\theta}) = \bar{\theta} - \hat{\theta}e(\hat{\theta})$. When the agent reports $\hat{\theta} = \theta_L$ ($\hat{\theta} = \theta_H$) and the principal observes a cost C_L (C_H), the principal pays the agent a transfer t_L (t_H) and recommends to exert effort e_L (e_H). Let $\Delta\theta = (\theta_L/\theta_H)^2 < 1$ and $R = 1 - \Delta\theta < 1$. A feasible contract to the agent must satisfy the individual rationality (IR) and incentive compatibility (IC) constraints

$$IR(L): \; t_L \geq e_L^2/2 \qquad IC(L): \; t_L - e_L^2/2 \geq t_H - e_H^2/2\Delta\theta$$

$$IR(H): \; t_H \geq e_H^2/2 \qquad IC(H): \; t_H - e_H^2/2 \geq t_L - e_L^2\Delta\theta/2$$

A standard result is that when the constraints IR(L) and IC(H) are binding, IC(L) and IR(H) are not binding (the proof is standard and hence omitted). The principal's problem with the binding constraints IR(L) and IC(H) is to choose e_L and e_H to maximize

$$V - \bar{\theta} + q\left[\theta_H e_H - \frac{e_H^2}{2} - R\frac{e_L^2}{2}\right] + (1-q)\left[\theta_L e_L - \frac{e_L^2}{2}\right]$$

The solution to this (No-Monitor) problem and the principal's utility are:

$$e_L^{NM} = \frac{(1-q)\theta_L}{(1-q)+qR} \qquad e_H^{NM} = \theta_H$$

$$t_L^{NM} = \frac{e_L^2}{2} \qquad t_H^{NM} = \frac{e_H^2}{2} + R\frac{e_L^2}{2}$$

$$EU_P^{NM} = V - \bar{\theta} + q\frac{\theta_H^2}{2} + \frac{(1-q)^2\theta_L^2}{2\left[(1-q)+qR\right]} \tag{3.1}$$

In order to elicit high effort from the high-productivity agent (who has incentives to claim that she is inefficient), the principal pays her an information rent. Eliciting lower effort from the low-productivity agent reduces this rent, which depends on the high-productivity agent's gains from misreporting.

To avoid paying rents, the principal may offer a No-Rent contract to be accepted only by the high-productivity agent: $t = \theta_H^2/2$ if the observed production cost is $C_H = \bar{\theta} - \theta_H^2$, and nothing otherwise. The principal's utility is $EU_P^{NR} = q\left\{V - \bar{\theta} + \theta_H^2/2\right\}$. We assume parameters such that the principal's utility under the No-Monitor contract is higher than that under the No-Rent contract (it is sufficient to assume a high V).

3.3.2 No Collusion

The monitor's signal adds new information about the agent's type that can be used in the main contract. We solve for the contract with a supervisor or an auditor separately, and show the conditions such that the principal chooses either of them.[12]

Contract with a Supervisor

Before the agent exerts effort, the supervisor reports his signal $r = \sigma$ to the principal, who uses it to contract agent's effort and wage. Let the agent's effort be e_{jr} and her compensation be t_{jr} when she reports $\hat{\theta} \in \{\theta_L, \theta_H\}$ and the monitor reports $r \in \{0, L, H\}$ in a direct mechanism. The constraints for a feasible contract are

$$\begin{aligned} &\text{IR(jr)}: \quad t_{jr} \geq e_{jr}^2/2 \\ &\text{IC(Hr)}: \quad t_{Hr} - e_{Hr}^2/2 \geq t_{Lr} - e_{Lr}^2 \Delta\theta/2 \qquad \text{for } j \in \{L, H\},\ r \in \{0, L, H\} \\ &\text{IC(Lr)}: \quad t_{Lr} - e_{Lr}^2/2 \geq t_{Hr} - e_{Hr}^2/2\Delta\theta \end{aligned} \tag{3.2}$$

Let π_{jr} denote the probability of occurrence of each state, where $j \in \{L, H\}$ and

[12] In some cases, the optimal contract for a given monitor has been analyzed elsewhere and we compute it with the assumptions made here. We acknowledge the original author in each case.

$r \in \{0, L, H\}$.[13] Since collusion is not possible the principal sets $w_{jr} = 0$. As in the No-Monitor contract, if constraints IR(Lr) and IC(Hr) are binding, the other constraints are non-binding. The principal's problem with the binding constraints is to choose $\{e_{jr}\}$ to maximize

$$V - \bar{\theta} + \sum_{r \in \{0,L,H\}} \pi_{Hr} \left\{ \theta_H e_{Hr} - \frac{e_{Hr}^2}{2} - R\frac{e_{Lr}^2}{2} \right\} + \sum_{r \in \{0,L,H\}} \pi_{Lr} \left\{ \theta_L e_{Lr} - \frac{e_{Lr}^2}{2} \right\} \quad (3.3)$$

The optimal effort and compensation are:

$$\begin{aligned} e_{H0} &= e_{HL} = e_{HH} = \theta_H & e_{L0} &= \frac{(1-q)\theta_L}{(1-q)+qR} \\ e_{LL} &= \frac{(1-q)\alpha\theta_L}{(1-q)\alpha + q(1-\alpha)R} & e_{LH} &= \frac{(1-q)(1-\alpha)\theta_L}{(1-q)(1-\alpha)+q\alpha R} \end{aligned} \quad (3.4)$$

$$t_{Lr} = \frac{e_{Lr}^2}{2}, \quad t_{Hr} = \frac{\theta_H^2}{2} + R\frac{e_{Lr}^2}{2}, \quad w_{jr} = 0, \qquad j \in \{L, H\} \text{ and } r \in \{0, L, H\} \quad (3.5)$$

The next Proposition summarizes this result (see Tirole [56]).

Proposition 9 *The optimal contract when the principal hires a supervisor and side-contracts cannot be enforced satisfies (3.4)-(3.5). The supervisor is hired always.*

We can observe the benefits of flexibility in contracting from equations (3.4)-(3.5). The principal pays no rent to the type-θ_L agent for any monitor's report, but cannot eliminate the rents to the type-θ_H agent. Let the agent's rents in state Hr be $R_{Hr} = Re_{Lr}^2/2$. The optimal contract is such that $e_{LL} > e_{L0} > e_{LH}$ and

[13] There are six states whose probabilities are $\pi_{L0} = (1-q)(1-p)$, $\pi_{LL} = (1-q)p\alpha$, $\pi_{LH} = (1-q)p(1-\alpha)$, $\pi_{H0} = q(1-p)$, $\pi_{HL} = qp(1-\alpha)$, and $\pi_{HH} = qp\alpha$. We will use this simplification throughout this chapter.

$R_{HL} > R_{H0} > R_{HH}$. By obtaining the signal before the agent exerts effort, the principal finds it profitable to create higher distortions in the low-probability state LH (she reduces e_{LH}), which allows her to pay lower rents R_{HH} in the high-probability state HH. Also, the principal reduces distortions in the high-probability state LL (she increases e_{LL}), which leads to higher rents R_{HL} in the low-probability state HL. In the limiting case of perfectly informative signal ($\alpha = 1$), the high inefficiencies and rents are ex ante costless (states LH and HL have probability 0).

Contract with an Auditor

The principal can use the report from an auditor only to contract compensations since the agent already exerted her effort. From Revelation Principle, the principal can relate a low production cost (or high output) to a type-θ_H agent. In this case there is no need to perform an audit, and the agent is paid t_h. When the production cost is high (or output is low), the principal cannot infer whether this is because the type-θ_L agent has exerted the right effort or the type-θ_H agent has shirked (the agent is paid t_l). In this case the principal sends the auditor (with probability δ) and penalizes the agent when the auditor's report does not match with the agent's type report. Fines are z^0 if $r = 0$ and z^H if $r = H$. Using these results, the agent's participation and incentive constraints are

$$
\begin{aligned}
&\text{IR(L)}: \quad t_l - \delta\left[(1-p)z^0 + p(1-\alpha)z^H\right] \geq e_l^2/2 \\
&\text{IC(L)}: \quad t_l - \delta\left[(1-p)z^0 + p(1-\alpha)z^H\right] - e_l^2/2 \geq t_h - e_h^2/2\Delta\theta \\
&\text{IR(H)}: \quad t_h \geq e_h^2/2 \\
&\text{IC(H)}: \quad t_h - e_h^2/2 \geq t_l - \delta\left[(1-p)z^0 + p\alpha z^H\right] - e_l^2\Delta\theta/2
\end{aligned}
\tag{3.6}
$$

The principal pays $w^r = 0$ to the auditor for any report, and sends him with probability $\delta = 1$. Let $-_{ep} = \{e_h, e_l, t_h, t_l, z^0, z^H\}$ be the set of choice variables. The principal's problem is to choose $-_{ep}$ to maximize

$$
V - \bar{\theta} + q\left\{\theta_H e_h - t_h\right\} + (1-q)\left\{\theta_L e_l - t_l + \left[(1-p)\,z^0 + p\,(1-\alpha)\,z^H\right]\right\} \tag{3.7}
$$

subject to constraints (3.6) and the limited liability constraints z^0, $z^H \leq z$; w^0, w^L, $w^H \geq 0$. Let α_1^* denote the value of α such that IR(H) is non-binding for $\alpha < \alpha_1^*$, and α_2^* the value of α such that IC(H) is non-binding for $\alpha > \alpha_2^*$ (from equations (3.27) and (3.28) in the Appendix, respectively), where

$$
\alpha_1^* = \frac{1}{2} + \frac{(1-q)^2\theta_L^2 R}{4pz\left[(1-q)+qR\right]^2} \qquad \alpha_2^* = \frac{1}{2} + \frac{\theta_L^2 R}{2pz}
$$

The optimal effort and compensation are (see Baron and Besanko [2]):

$$
\begin{array}{llll}
 & \alpha < \alpha_1^* & \alpha_1^* \leq \alpha \leq \alpha_2^* & \alpha_2^* < \alpha \\
e_l: & \dfrac{(1-q)\theta_L}{(1-q)+qR} & \sqrt{\dfrac{2p(2\alpha-1)z}{R}} & \theta_L \\
t_h: & \dfrac{\theta_H^2}{2} + \dfrac{e_l^2 R}{2} - p(2\alpha-1)z & \dfrac{\theta_H^2}{2} & \dfrac{\theta_H^2}{2}
\end{array}
\tag{3.8}
$$

$$
e_h = \theta_H, \qquad t_l = \frac{e_l^2}{2} + (1-p\alpha)z, \qquad z^0 = z^H = z, \qquad w^0 = w^L = w^H = 0 \tag{3.9}
$$

The next Proposition summarizes the optimal contract.

Proposition 10 *The optimal contract when the principal hires an auditor and side-contracts cannot be enforced satisfies (3.8)-(3.9). The auditor is hired always.*

Proof: See Appendix.

This contract is less flexible than the contract with a supervisor since the agent's effort is not affected directly by the auditor's report. This report is used to punish the agent (which affects the agent's net compensation) when the principal obtains no favorable information about the agent's type. In particular, for a given informativeness α of the monitor's signal, the threat of punishment is enough to deter the type-θ_H agent from misreporting when the liability bound is high enough, and the principal achieves first-best utility.[14] This result is summarized in the next Corollary.

Corollary 2 *For any degree informativeness of the signal α, the principal achieves first-best utility when the punishment bound z is relatively high.*

Proof: Fix a value of α. The principal achieves first-best utility if $\alpha > \alpha_2^*$, which is satisfied when $z > z_B$, and $z_B = \theta_L^2 R/p(2\alpha - 1)$. *Q.E.D.*

Optimal Timing without Collusion

In this section we discuss the principal's monitoring timing decision when side contracts between the agent and the monitor cannot be enforced. When the principal

[14] Note, however, that the principal punishes the type-θ_L agent in equilibrium. When the type-θ_H agent reports truthfully, she is not audited (see Kofman and Lawarrée [38]). This result of punishment in equilibrium is common in the literature on collusion under uncertainty (see, for example, Green and Porter [24]), in which the colluding parties use a self-policing instrument that triggers when a bad outcome occurs (which can be caused by nature or by other party's deviation).

hires a supervisor, she finds profitable to distort allocations and minimize the agent's expected rents depending on the supervisor's signal. On the other hand, the auditor's signal cannot be used to modify allocations, but can be used to punish the agent.

Not surprisingly, the information of an auditor is very useful to the principal when the latter can punish the agent strongly. As we show in the Appendix (Proof of Theorem 1), there exists a minimum liability bound $\bar{z}$ such that an auditor is optimal for $z > \bar{z}$ for any precision of his signal.[15]

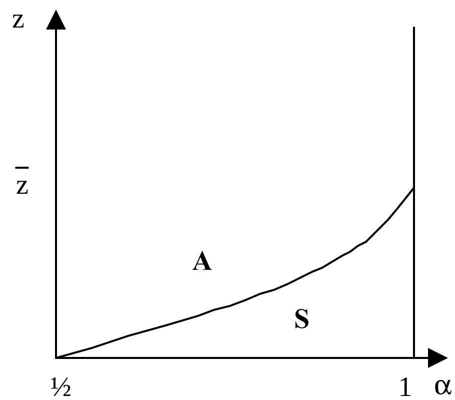
For a given punishment instrument $z < \bar{z}$, the principal hires the monitor to supervise the agent when his signal is informative of the agent's type, and to audit the agent when his signal is noisy (See Figure 3.1). In this case, the principal is able to reduce the type-θ_H agent's rent when the supervisor observes a "correct" signal (that is, $\sigma = H$, which is more probable when α is high). The next Theorem emphasizes the importance of monitoring timing in absence of collusion.

Theorem 1 *Suppose that side-contracts between the agent and the monitor cannot be enforced. Auditing is optimal when the principal's punishment instrument is strong (z is relatively high), or when it is weak and the monitor's signal of the agent's type is noisy (low α). Supervising is optimal for a weak punishment instrument (low z) and an informative signal (high α).*

Proof: See Appendix.

[15] In a paper on Law enforcement, Shavell [53] shows that the availability of harm-based sanctions is an important determinant of the (ex post) legal intervention stage.

Figure 3.1: Optimal Timing: Non-Enforceable Side Contracts



Suppose now that the principal is constrained to monitor the agent's type, so that the problem is when to obtain information that is available at the outset. Another interpretation of Theorem 1 is that the principal can strategically delay the gathering of relevant information. In particular, this is so when the punishment instrument is sufficient to deter the agent. As we show later, these results become stronger with collusion under non-forgeable information (although the main trade-off is not eliminated), but may revert when information is forgeable or soft.

Next, we present the principal's response to change in parameters. Assume that the liability bound z is such that $z < \bar{z}$, so that there exists a cut-off $\alpha^C \in (1/2, 1]$ of the monitor's signal informativeness for which a supervisor is optimal when $\alpha > \alpha^C$ and an auditor is optimal otherwise. Then, we have the following

Result 1 *Consider as reference the informativeness of the monitor's signal α.*

- *The cut-off α^C increases in z. Auditing is optimal for a broader region of the informativeness of the monitor's signal as the liability bound increases.*
- *The cut-off α^C decreases in R. The region of optimality of a supervisor expands out as the adverse selection problem is more severe.*[16]

Proof: See Appendix.

The first result is a direct implication of Theorem 1. The second result is a consequence of the way the principal designs the contract. The supervisor's signal is useful to reduce the agent's rent in the more probable state ($R_{HH} < R_{H0}$). The auditor's signal is only useful to punish the agent (whose rents are the rents under the No-Monitor contract net of the expected punishment, see t_h in (3.8)). When the type distribution is more disperse, which implies a more severe adverse selection problem (measured by a higher R), the first contract is more suitable to control the agent's rent. For a given α and z, the decrease in the principal's utility with a supervisor is lower than that in the utility with an auditor, and the intersection occurs at a lower α. Therefore, the region of optimality of a supervisor expands out.

3.4 Collusion: Hard and Non-Forgeable Information

In the remaining of the chapter we allow for collusion between the agent and the monitor. In this section, we assume that the monitor's information is hard and non-

[16] This result applies to the case in which the agent still earns some rent when audited (i.e., parameters are such $\alpha_1^* \geq 1$ in equation (3.8)), and for $q \leq 1/(1+R)$.

forgeable, so the monitor discretion lies in concealing his signal from the principal. We also assume that the monitor has all the bargaining power when he bargains with the agent over a side transfer, and hence makes a take-it-or-leave-it offer to the agent. Side transfers may be inefficient: a transfer b from the agent is valued $v(b) = kb$ by the monitor (with $0 \leq k \leq 1$).

3.4.1 Contract with a Supervisor

The agent's participation and incentive compatibility constraints in a feasible contract are (3.2). The supervisor's participation and limited liability constraints are

$$w_{jr} \geq 0, \qquad \text{for } j \in \{L, H\}, \quad r \in \{0, L, H\}^{17} \tag{3.10}$$

The supervisor's discretion lies in concealing his signal. To avoid collusion, the principal has to compensate the coalition so that concealment of the signal is not profitable for the coalition members. Let the agent's rent be $R_{jr} = t_{jr} - e_{jr}^2/2$. A feasible contract must also satisfy the *simplified coalition constraints*

$$\begin{aligned} \text{CC(HH)}: \quad & w_{HH} + kR_{HH} \geq w_{H0} + kR_{H0} \\ \text{CC(HL)}: \quad & w_{HL} + kR_{HL} \geq w_{H0} + kR_{H0} \end{aligned} \tag{3.11}$$

The coalition constraints are (3.33) in the Appendix given that the monitor does not know the agent's type at the side-contract stage. As we show in the Appendix, these conditions simplify to (3.11) when the IR(Lr) and IC(Hr) constraints are binding

[17] The supervisor's compensation must be non-negative (limited liability). Also, it must satisfy his participation constraint, which is $qw_{Hr} + (1-q)w_{Lr} \geq 0$, for $r \in \{0, L, H\}$ (since the supervisor does not know the agent's type). These constraints are satisfied when wages are non-negative.

(which it will be true in an optimal contract). The simplified coalition constraints are *as if* the monitor knew that the agent's type is θ_H when offering a side-contract to the agent.[18]

As we defined in footnote (13), the probabilities of occurrence of each state are π_{jr}. The principal's problem is to choose effort and compensations to maximize

$$V - \bar{\theta} + \sum_{j \in \{L,H\}, r \in \{0,L,H\}} \pi_{jr} \{\theta_j e_{jr} - t_{jr} - w_{jr}\} \tag{3.12}$$

subject to (3.2), (3.10) and (3.11). Define α_h such that constraint CC(HH) is binding with $w_{HH} > 0$ for $\alpha > \alpha_h$, where

$$\alpha_h = \begin{cases} \frac{-[2(1-p)-k] + \sqrt{[2(1-p)-k]^2 + 4pk(1-p)}}{2pk} & \text{if} \quad k > 0 \\ 1/2 & \text{if} \quad k = 0 \end{cases} \tag{3.13}$$

The optimal effort and wage are

$$\begin{array}{llll} & \alpha \leq \alpha_h,\ p < 1 & \alpha > \alpha_h,\ p < 1 & p = 1 \\ e_{L0}: & \frac{(1-q)(1-p\alpha)\theta_L}{(1-q)(1-p\alpha)+qR(1-p(1-\alpha))} & \frac{(1-q)(1-p)\theta_L}{(1-q)(1-p)+qR[1-p(1-\alpha k)]} & - \\ e_{LL}: & \frac{(1-q)\alpha\theta_L}{(1-q)\alpha+q(1-\alpha)R} & \frac{(1-q)\alpha\theta_L}{(1-q)\alpha+q(1-\alpha)R} & \frac{(1-q)\alpha\theta_L}{(1-q)\alpha+q(1-\alpha)R} \\ e_{LH}: & e_{L0} & \frac{(1-q)(1-\alpha)\theta_L}{(1-q)(1-\alpha)+q\alpha R(1-k)} & \frac{(1-q)(1-\alpha)\theta_L}{(1-q)(1-\alpha)+q\alpha R} \end{array} \tag{3.14}$$

[18] In Tirole [56], the supervisor knows the agent's type when he obtains a signal. This corresponds to $\alpha = 1$ in our model.

$$e_{H0} = e_{HL} = e_{HH} = \theta_H$$

$$t_{Lr} = \frac{e_{Lr}^2}{2}, \qquad t_{Hr} = \frac{\theta_H^2}{2} + R\frac{e_{Lr}^2}{2}, \qquad w_{Lr} = 0 \qquad r \in \{0, L, H\} \tag{3.15}$$

$$w_{H0} = w_{HL} = 0, \qquad\qquad w_{HH} = kR\left(e_{L0}^2 - e_{LH}^2\right)/2$$

The proof of Proposition 11 shows this result, together with the conditions for an optimal contract (Tirole [56]).

Proposition 11 *The optimal contract when the principal hires a supervisor with hard and non-forgeable information satisfies (3.14)-(3.15). The supervisor is always hired.*

Proof: See Appendix.

Take the contract under no collusion ((3.4)-(3.5)) as benchmark to analyze how the principal changes it when the agent-supervisor coalition can conceal the supervisor's information. The type-θ_H agent's rent when $\sigma = L$ is higher than that when $\sigma = 0$. Hence the principal does not change e_{LL} (from its value in the no-collusion contract) since concealment of the supervisor's signal is not profitable to the agent.

When the signal is $\sigma = H$, the type-θ_H agent will bribe the monitor to hide his signal. The principal has two ways to avoid it and obtain the true signal. The cheaper way is to compensate the monitor with the *adjusted* agent's rent (which is $w_{HH} = k(R_{H0} - R_{HH})$). The "coalition rent" is $R_{HH} + w_{HH} = kR_{H0} + (1-k)R_{HH}$ in state HH, and R_{H0} in state H0. The agent's rent under a signal $\sigma = 0$ spreads over state HH, while her rent under a signal $\sigma = H$ is lowered (to a proportion

$1 - k$). Therefore, the principal reduces e_{L0} and increases e_{LH} (from the values in the no-collusion contract). This option is profitable when the supervisor's signal is accurate ($\alpha > \alpha_h$). The more expensive way is not to distort allocations in the states under conflict and set $e_{L0} = e_{LH}$ (and hence $R_{H0} = R_{HH}$). This option is profitable when the supervisor's signal is noisy ($\alpha \leq \alpha_h$). Nevertheless, the supervisor is always valuable to the principal.

From equation (3.13), the cut-off value α_h increases in k (in particular, $\alpha_h = 1/2$ if $k = 0$, and $\alpha_h = 1$ if $k = 1$). When $\alpha \leq \alpha_h$, allocations are not modified as k increases. When $\alpha > \alpha_h$, the principal either pays a higher coalition rent or switches to the more expensive option as the side transfers are more efficient. Therefore, the principal's utility is non-increasing in k. Moreover, this utility is lower than that under no collusion for any $p < 1$.[19] This result is summarized next.

Corollary 3 *When the principal hires a supervisor, collusion imposes costs on the optimal contract. In particular, the principal's utility is lower than that under no collusion, and is non-increasing in the side-transfer efficiency parameter k.*

3.4.2 Contract with an Auditor

As in Section 3.2.2, an audit is performed when the production cost is high, which is incurred by a type-θ_L agent (but it can be incurred by a deviant type-θ_H agent).

[19] This can be checked by comparing efforts and compensations (3.4)-(3.5) and (3.14)-(3.15). In the limit case of $p = 1$, collusion is not a threat since the supervisor cannot claim having not seen anything, and the solution is the no-collusion contract.

The principal punishes the agent when the auditor finds unfavorable information to the agent. In those cases, the type-θ_L agent is willing to bribe the auditor in order to change the latter's report $r = H$ to $r = 0$. The additional *simplified* constraint for a collusion-proof contract is:[20]

$$\text{CC(LH)}: \qquad w^H - kz^H \geq w^0 - kz^0 \tag{3.16}$$

But the contract (3.8)-(3.9) is already designed such that the agent has no incentive to bribe the monitor (since $z^H = z^0$ and $w^H = w^0$), and hence eliminates all stakes in collusion. Hence, we have the following result (Kessler [33] shows this result for effort monitoring):

Corollary 4 *When the principal hires an auditor with hard and non-forgeable information, collusion imposes no cost to the principal.*

3.4.3 Optimal Timing

Corollaries 3 and 4 provide the necessary information to analyze the effects of collusion on the principal's timing decision. When the principal hires a supervisor, she can still use the signal to achieve some flexibility in contracting. Of course, this flexibility either comes at a cost of paying the supervisor to report his signal truthfully or is limited to avoid concealing the signal. When the principal hires an auditor, she avoids collusion at no cost. Then we have the following

[20] The auditor does not know the agent's type at the side-contracting stage. Using similar techniques as those to prove condition (3.11), the simplified coalition constraint is (3.16).

Proposition 12 *When the principal hires a monitor with hard and non-forgeable information:*

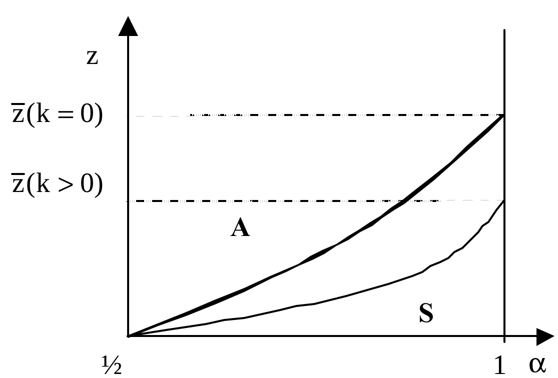
(i) Theorem 1 applies when the side transfers are inefficient (low k). That is,

- *Auditing is optimal when the punishment instrument is strong (high z), or when the punishment is weak and the signal is noisy (low α).*

- *Supervising is optimal when the punishment instrument is weak (low z) and the signal is precise (high α).*

(ii) The region of optimality of a supervisor shrinks as side transfers are more efficient (k increases).

Figure 3.2: Optimal Timing: Collusion with Hard and Non-Forgeable Information



As side transfers are more efficient (k is higher) the region of optimality of a supervision narrows down.

The optimal timing results from Section 3.3.2 hold here when side transfers are inefficient (k is low), since the cost of avoiding collusion (captured in k) is low. As side transfers become more efficient, an auditor is more likely to be preferred. Note, however, that the principal will hire the monitor to supervise the agent when his signal is informative and punishment is weak, even under efficient side transfers (see Figure 3.2).

3.5 Hard and Forgeable Information

Now we turn to the case in which the agent-monitor coalition can manipulate the monitor's report. It is straightforward to note that the contracts in Sections 3.3.2 and 3.4 are no longer optimal. Whenever the agent-monitor coalition is paid higher in one state of the world, the parties can coordinate a change in reports.

3.5.1 Contract with a Supervisor

The (simplified) coalition constraints in a feasible contract are as follows. From Section 3.3.2, the principal sets $e_{LL} > e_{L0} > e_{LH}$ and hence $R_{HL} > R_{H0} > R_{HH}$, where R_{Hr} is the agent's rent in state Hr, for $r \in \{0, L, H\}$ (see (3.4)-(3.5)). Hence a type-θ_H agent will bribe the supervisor to change his report from 0 or H to L. The principal eliminates the coalition benefits to forge the signal if:[21]

$$\begin{aligned} \text{CC(H0-HL)}: \quad & w_{H0} + kR_{H0} \geq w_{HL} + R_{HL} \\ \text{CC(HH-HL)}: \quad & w_{HH} + kR_{HH} \geq w_{HL} + R_{HL} \end{aligned} \tag{3.17}$$

Define α_1^f such that constraint CC(HH-HL) is binding with $w_{HH} > 0$ for $\alpha > \alpha_1^f$, and α_2^f such that constraint CC(H0-HL) is binding with $w_{H0} > 0$ for $\alpha > \alpha_2^f$ (equations (3.36) and (3.35) in the Appendix, respectively):

$$\alpha_1^f = \frac{1}{2-k} \qquad \alpha_2^f = \frac{k + (1-k)p}{2p(1-k)} \tag{3.18}$$

The optimal effort and compensation are

$$\begin{array}{llll} & \alpha \leq \alpha_1^f & \alpha_1^f < \alpha \leq \alpha_2^f & \alpha_2^f < \alpha < 1 \\ e_{L0}: & \frac{(1-q)\theta_L}{(1-q)+qR} & \frac{(1-q)(1-p(1-\alpha))\theta_L}{(1-q)(1-p(1-\alpha))+qR[(1-p\alpha(1-k)]} & \frac{(1-q)\theta_L}{(1-q)+qR(1-k)} \\ e_{LL}: & e_{L0} & e_{L0} & \frac{(1-q)p\alpha\theta_L}{(1-q)p\alpha+qR[k+p(1-\alpha)(1-k)]} \\ e_{LH}: & e_{L0} & \frac{(1-q)(1-\alpha)\theta_L}{(1-q)(1-\alpha)+q\alpha R(1-k)} & \frac{(1-q)(1-\alpha)\theta_L}{(1-q)(1-\alpha)+q\alpha R(1-k)} \end{array} \tag{3.19}$$

$$\begin{aligned} & e_{Hr} = \theta_H, \quad t_{Lr} = \frac{e_{Lr}^2}{2}, \quad t_{Hr} = \frac{\theta_H^2}{2} + R\frac{e_{Lr}^2}{2}, \quad w_{Lr} = 0, \quad r \in \{0, L, H\} \\ & w_{HL} = 0, \quad w_{H0} = k(t_{HL} - t_{H0}), \quad w_{HH} = k(t_{HL} - t_{HH}) \end{aligned} \tag{3.20}$$

The next Proposition summarizes the optimal contract.

Proposition 13 *The optimal contract when the principal hires a supervisor with hard and forgeable information satisfies (3.19)-(3.20).*

[21] Conditions (3.17) are derived using similar techniques as those used to prove conditions (3.11).

Proof: See Appendix.

First, let us interpret the coalition constraints. The agent's rent in state Hr is $R_{Hr} = Re_{Lr}^2/2$. Suppose that the principal offers a contract such that $R_{H0} < R_{HL}$ and $w_{H0} = w_{HL}$. The coalition will change the supervisor's report to $r = L$. On the other hand, if $R_{H0} > R_{HL}$, the coalition will change the report to $r = 0$. When the principal avoids collusion in one direction, she creates stakes in collusion in the other direction. In general, an optimal contract must have

$$w_{H0} + kR_{H0} = w_{HL} + kR_{HL} = w_{HH} + kR_{HH}$$

This contract, which is designed to avoid changing the supervisor's report to $r = L$, also prevents coalition deviations in any direction (e.g., in state HH there will not be incentives to claim that the state is H0, etc.).

The contract (3.19)-(3.20) shows that the supervisor's signal must be of some minimum accuracy for the principal to hire him. Two reasons explain this result. First, a noisy signal is not very informative of the agent's type. Second, when the supervisor's report can be manipulated, the information value of the signal is reduced. Hence, we have the following

Corollary 5 *When information is hard and forgeable, the supervisor is hired if his signal exceeds a minimum degree of informativeness (i.e,* $\alpha > \alpha_1^f$*).*

Contracting is more flexible as the signal becomes more informative of the agent's type. The principal only distorts effort e_{LH} (from $e_{L0} = e_{LL}$) for $\alpha_1^h < \alpha \leq \alpha_2^h$, while she sets $e_{LL} > e_{L0} > e_{LH}$ when $\alpha > \alpha_2^h$.

Hiring a supervisor is profitable to the principal when side transfers are inefficient. In the (opposite) limiting case of efficient side transfers ($k = 1$), the principal has to transfer the whole agent's rent to the supervisor to avoid forgery, and hence deals with the agent directly for any precision of the supervisor's signal (note that $\alpha_1^f(k = 1) = 1$).

Finally, when $\alpha = 1$ (not included in (3.19)-(3.20)), forgery is not possible (but concealment is) because the supervisor, who observes the agent's type correctly, cannot claim having seen $\sigma = L$ ($\sigma = H$) when the agent reports θ_H (θ_L) truthfully, but can claim having observed $\sigma = 0$. In this case, the contract is (3.14)-(3.15).

3.5.2 Contract with an Auditor

As it is shown in Section 3.3.1 (no collusion), the principal audits the agent when the observed cost is high, which is incurred by a type-θ_L agent (but it can be incurred by a deviant type-θ_H agent). The principal does not punish the type-θ_L agent when the auditor's report is $r = L$, which leads the agent to bribe the auditor to make up any report to $r = L$.[22] Therefore, the (simplified) coalition constraints are

$$\begin{aligned} \text{CC(0L)}: &\quad w^0 - kz^0 \geq w^L \\ \text{CC(HL)}: &\quad w^H - kz^H \geq w^L \end{aligned} \tag{3.21}$$

We solve for the optimal contract in the Appendix. Consider the three cut-offs α_1 (an auditor is not hired for $\alpha < \alpha_1$, see (3.40) in the Appendix), α_2 (the type-θ_H agent

[22] Again, (3.21) is obtained with similar techniques as those used to prove conditions (3.11).

earns positive rent for $\alpha < \alpha_2$, see (3.41)) and α_3, (an auditor is sent with probability less than 1 for $\alpha > \alpha_3$, see (3.42)). We repeat α_1 (which determines whether the auditor is hired or not) for convenience.

$$\alpha_1 = \frac{q + (1-q)k}{2q + (1-q)k} \tag{3.22}$$

The optimal contract satisfies (see Kofman and Lawarrée [38] for the limiting case of $p = 1$):[23]

$$\begin{array}{lllll} & \alpha < \alpha_1 & \alpha_1 \le \alpha < \alpha_2 & \alpha_2 \le \alpha \le \alpha_3 & \alpha_3 < \alpha \quad (\delta \le 1) \\ e_l : & \frac{(1-q)\theta_L}{(1-q)+qR} & \frac{(1-q)\theta_L}{(1-q)+qR} & \sqrt{\frac{2p(2\alpha-1)z}{R}} & \frac{(2\alpha-1)\theta_L}{(2\alpha-1)+(1-\alpha)Rk} \\ t_l : & \frac{e_l^2}{2} & \frac{e_l^2}{2} + p(1-\alpha)z & \frac{e_l^2}{2} + p(1-\alpha)z & \frac{e_l^2}{2} + p\delta(1-\alpha)z \\ t_h : & \frac{\theta_H^2}{2} + R\frac{e_l^2}{2} & \frac{\theta_H^2}{2} + R\frac{e_l^2}{2} - p(2\alpha-1)z & \frac{\theta_H^2}{2} & \frac{\theta_H^2}{2} \end{array} \tag{3.23}$$

$$e_h = \theta_H, \quad w^0 = w^L = 0, \quad w^H = kz^H, \quad z^0 = 0, \quad z^H = z \text{ if } \alpha > \alpha_1 \tag{3.24}$$

This result is summarized in the next Proposition.

Proposition 14 *The optimal contract when the principal hires an auditor with hard and forgeable information satisfies (3.23)-(3.24).*

Proof: See Appendix.

The principal designs compensations such that $w^0 - kz^0 = w^L = w^H - kz^H$. Therefore, the coalition finds unprofitable to forge the signal in any direction. In particular,

[23] When the principal hires an auditor and pays him a positive wage, she faces a commitment problem after the outcome is realized. Once the agent makes a truthful report, the incentive for the principal to send an auditor vanishes. We assume that the principal created some reputation to hold her word (see Khalil and Lawarré [34]). This commitment issue does not arise when the principal hires a supervisor.

the principal punishes the type-θ_L agent when she "infers" a high productivity. This occurs when the monitor's report is $r = H$. To obtain a truthful report, the principal compensates the auditor with $w^H = kz^H$.

Opposite to the case with a supervisor, the principal may still hire the auditor when side transfers are efficient ($\alpha_1(k = 1) < 1$). But as in that case, the auditor is not hired when his signal is noisy ($\alpha_1 > 1/2$). As before, manipulation of information makes the noisy signal less valuable. We have the following

Corollary 6 *The principal contracts with the agent directly when the auditor's signal is noisy (i.e., $\alpha < \alpha_1$).*

As the signal is more accurate, the punishment instrument is used to reduce the agent's rent. For certain combination of signal accuracy and punishment (which corresponds to $\alpha > \alpha_3$), the punishment is sufficiently deterrent to extract the type-θ_H agent's rent and obtain an honest report from him. In this case, the principal sends the auditor with probability less than 1, which reduces the expected compensation to the auditor.

3.5.3 Optimal Timing

In this section we discuss how forgery of the monitor's signal affects the principal's timing decision. For low penalties, a supervisor is more profitable to the principal when he obtains the "right" information about the agent's type. The contract (3.19)-(3.20) is designed such that the type-θ_H agent's rent is reduced in the high-probability state HH (in particular, $R_{HH} \leq R_{H0} \leq R_{HL}$), while the type-$\theta_L$ agent's effort increases in the high-probability state LL ($e_{LL} \geq e_{L0} \geq e_{LH}$). When information is forgeable, stakes in collusion are created because the agent will bribe the supervisor to change a report $r \in \{0, H\}$ to $r = L$. The principal must compensate the supervisor in states H0 and HH to obtain a truthful report (or, eventually, she does not distort efforts). The cost of controlling collusion increases partially (compared to the case of information concealment).

On the other hand, the contract with an auditor (3.23)-(3.24) is designed such that the type-θ_L agent is punished when the auditor's report is $r = H$ (i.e., when he observes the wrong signal). The auditor must be compensated to report truthfully, for he will be bribed by the agent to report $r = L$ otherwise. The threat of collusion imposes positive cost (controlling collusion is costless when information can only be concealed), which is higher when the signal is noisier. The relevant trade-off under hard and forgeable information is flexibility in contracting with additional collusion costs vs. punishment with positive collusion costs, and the optimal timing decision may differ from that under hard and non-forgeable information (in Section 4). The next Theorem summarizes the main results (the results for $\alpha_1^f < \alpha_1$ are represented in Figure 3.3).[24]

Theorem 2 *Suppose that the monitor's information is hard and forgeable.*

(i) When parameters are such that $\alpha_1^f < \alpha_1$ *(from (3.18) and (3.22), respectively):*

[24] The values $\bar{z}_1$ and $\bar{z}_2$ are computed in the proof of Theorem 2.

- *Supervising is optimal for all* $\alpha \in [\alpha_1^f, 1]$ *when the punishment is weak* $(z < \bar{z}_1)$.

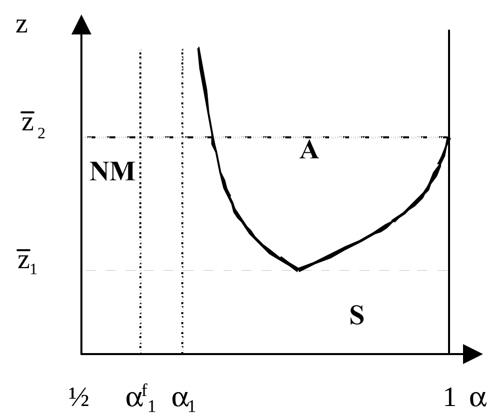
- *For intermediate levels of punishment* $(z \in [\bar{z}_1, \bar{z}_2])$, *there exist two cut-off values of* α, α_1^C *and* α_2^C, *such that auditing is optimal for* $\alpha_1^C \leq \alpha \leq \alpha_2^C$ *and supervising is optimal otherwise.*

- *When the punishment is strong* $(z > \bar{z}_2)$ *supervising is optimal for* $\alpha_1^f \leq \alpha < \alpha_1^C$ *and auditing is optimal for* $\alpha > \alpha_1^C$.

(ii) When parameters are such that $\alpha_1^f \geq \alpha_1$, α_1^C *does not exist. The results for* α_2^C *still hold.*

Proof: See Appendix.

Figure 3.3: Optimal Timing: Collusion with Hard and Forgeable Information



A: Auditor; S: Supervisor; NM: No Monitor.

The cut-off α_2^C plays the role of α^C in Sections 3.3.2 (optimal timing) and 3.4. In those sections we showed that a monitor is hired to supervise the agent when his signal is accurate (i.e., for $\alpha > \alpha^C$) and the punishment instrument is weak, and to audit the agent otherwise. In this section, we obtain the same result for high α.

From Corollaries 5 and 6, a monitor (supervisor or auditor) is hired if his signal exceeds some minimum level of informativeness of the agent's type (α_1^f and α_1, respectively). When information is forgeable, there is additional value of supervising when the signal is noisy. This happens when $\alpha_1^f < \alpha_1$. In this case, the auditor is too costly (the cost to avoid an agent's bribe is higher than the effect on the agent's incentives) that the principal would rather contract with the agent directly. But it is possible that the supervisor's signal is sufficiently informative for the principal to benefit from distorting allocations (and hence a flexible contract is optimal). Note that the existence of this region does not depend on the strength of the punishment instrument. Under these circumstances, *the supervisor is optimal even for unbounded penalties.*

Next, we discuss the effect of a change in the structural parameters on the optimal timing decision. Assume parameters such that there exist both α_1^C and α_2^C (that is, $\bar{z}_1 < z < \bar{z}_2$). Define the length of the interval $[\alpha_1^f, \alpha_1^C]$ as $S_1 = \alpha_1^C - \alpha_1^f$, and the length of the interval $[\alpha_2^C, 1]$ as $S_2 = 1 - \alpha_2^C$. We have the following

Result 2 *Consider as reference the informativeness of the monitor's signal α.*

- *An increase in the liability bound z decreases both S_1 and S_2. Therefore, an auditor* *is optimal for a broader range of α as the punishment is stronger.*

• *The intervals S_1 and S_2 are non-decreasing in the degree of adverse selection (R). Therefore, a supervisor is optimal for a broader range of α as R increases.*[25]

• *The effect of an improvement in the efficiency of side transfers k on S_1 is undetermined, but it decreases S_2. Auditing is more probable for high values of α.*

Proof: See Appendix.

The results for S_1 still hold if $z > \bar{z}_2$. When the punishment instrument is strong, an agent exposed to audits is more deterred from misreporting her type, while an agent under supervision is not affected. Therefore an auditor is more probable to be hired.

The agent's information rent increases as the adverse selection problem is more severe (higher R). The flexibility from a contract with a supervisor allows the principal to manipulate this rent depending on the supervisor's signal (while the contract with an auditor is very rigid in that sense). It is more probable that the principal will hire a supervisor (strictly, both S_1 and S_2 are non-decreasing in R).

Consider now an increase in the efficiency of side transfers (higher k). The responsiveness of the decision to hire a supervisor is lower (higher) than that of the decision to hire an auditor for low (high) k (compare α_1^f and α_1). We may expect S_1 to increase (decrease) for lower (higher) values of k. A supervisor is more profitable for low k, but he becomes relatively more expensive (compared to the auditor) as

[25] The proof is done assuming parameters such that $\alpha_1^f > \alpha_1$ (so that α_1^C exists), $\alpha_2 \geq 1$ (the agent receives some rent when audited), and $q \leq 1/(1+R)$.

collusion is more serious. In the limiting case of $k = 1$ the supervisor is not valuable at all, while the principal can use an auditor for $\alpha > 1/(1+q)$.

On the other hand, the effect of an increase in k on S_2 is more evident. The increase in the compensation to the auditor (which happens when he obtains a wrong signal) is very small when α is high. But the increase in the compensation to the supervisor is high since the latter is paid more when he obtains a correct signal. Therefore, an auditor is more probable to be hired if side transfers are more efficient, provided that his signal is very informative of the agent's type.

An interesting extension is to allow for the efficiency in side transfers to depend on the stakes in collusion. In particular, side transfers may be more inefficient as bribes involve higher amounts (they can be modeled as $k(b)$ with $k'(b) < 0$), which corresponds to the fact that it may be more difficult to hide higher bribes. In our model this means that $k_S > k_A$ (where S and A stand for supervisor and auditor, respectively) when the punishment is strong. On the other hand, when the liability bound is low compared to the agent's rents, the opposite may happen (i.e., $k_A > k_S$).[26] Our previous results become stronger under this generalization. We showed in Theorem 2 that auditing is more probable to be optimal as z increases. Consequently, given that side transfers are more inefficient with higher stakes, there are additional reasons to hire an auditor. On the other hand, when z is low, there are more reasons to hire a supervisor given that side transfers are more inefficient in this case.

[26] Note that this discussion is not relevant in the hard-and-non-forgeable-information case, since avoiding collusion is costless when the principal hires an auditor.

3.6 Soft Information

In this section we study how the principal's timing choice changes when the monitor's information is soft (i.e., he has no verifiable proof of his signal), in whose case any manipulation of information is possible. The principal is (expected to be) more constrained in the choice of contracts. Two main problems arise when the contract from Section 3.3.2 (no collusion) is offered to the agent and monitor. First, as in Section 3.5, the agent-monitor coalition will change reports when the parties find it profitable. Second, the supervisor will change a report unilaterally when there are gains from doing so.

The next Theorem states that the principal can achieve the hard-and-forgeable-information utility (from Section 3.5) in a Perfect Bayesian Equilibrium. Hence, all the discussion in that section can be applied to soft information.

Theorem 3 *When the monitor's signal is soft (i.e., he has no verifiable proof of it), the principal achieves the same utility as that when information is hard and forgeable.*
i) The optimal contract is (3.19)-(3.20) in the supervisor case and (3.23)-(3.24) in the auditor case.
ii) The optimal timing decision is as in Theorem 2.
iii) When $\alpha = 1$ the principal achieves the hard-and-non-forgeable-information utility.

Proof: See Appendix for Parts i) and ii). Part iii) is straightforward from Section 3.5. *Q.E.D.*

This ***utility equivalence*** result derives from the fact that the principal can control collusion at no additional cost (compared to the hard-and-forgeable-information case). Suppose that the principal offers a collusion-free contract (i.e., a contract that does not take collusion constraints into consideration). Then, the monitor-agent coalition has incentives to make up a profitable signal (and the agent provides the monitor with the relevant information to do it).[27] Even easier for the coalition parties, when information is soft, they will coordinate reports without any need to proof them. Moreover, since the agent knows the monitor's signal, the principal can use an agent's report of that signal in the mechanism. By minimizing compensations when the reports about the monitor's signal differ, the principal can eliminate unilateral changes in reports. Note that whether information is verifiable or not, in many circumstances the monitor is valuable to the principal (i.e., when the accuracy of his signal exceeds a minimum α_1^f or α_1, depending on the monitoring timing), who pays him accordingly to obtain a truthful report.

We mentioned in the Introduction that Baliga [1] finds an utility equivalence result between soft information and hard and non-forgeable information in Tirole's [58] model.[28] Theorem 3 generalizes this result by showing that the principal can achieve the utility with hard-and-forgeable-information (from Section 3.5). Baliga's

[27] Of course, this result depends on the assumption that the agent provides this information costlessly.

[28] This model assumes adverse selection and no cost-reducing effort. The monitor observes either the agent's true type or nothing (that is, $\alpha = 1$ and $0 < p < 1$). The monitor's "soft" signal is about the agent's high productivity.

result applies to the limiting case of a perfectly informative signal ($\alpha = 1$), in whose case the principal's utility under hard and forgeable information is the same as that under hard and non-forgeable information.

Some authors argue that supervising may be linked to "softer" information (see Dalton [12]). For example, Tirole says that "... The general observation is that it is usually hard to obtain information from intermediate levels of a hierarchy. Both Cozier and Dalton insist that very often common sense directs the controller to falsify his information to allow the monitored group to obtain better results..." (Tirole [56], p. 184). Under this assumption, the advantages of auditing the agent would be evident. But, as it is observed in many circumstances, auditing may lead to "forgeable" information. The controlled person may "point out" relevant information for the controller to provide proof of his report (see Kofman and Lawarrée [38]). As we have shown here, the consequences of these two information structures on the principal's decision are the same. Therefore, in this environment there are no gains of auditing employees with the hope of changing to a "harder" information environment if information can still be forged.

We have to alert about the robustness of the optimal contract under soft information (contrary to the case with hard information) to other coalition formations. With hard information, a coalition against the agent to change the monitor's signal cannot form since the agent also observes the monitor's signal. A coalition against the monitor is not profitable (he is who provides the additional report to the principal). However, under soft information, if the auditor and principal meet to change the auditor's report from L to H to punish the agent, there is no reason not to make it (and the agent cannot say that the signal was effectively L). We can assume in this case that the principal sets a punishment on herself when the auditor and agent's report of the monitor's signal differ (the Equilibrium in Proposition 3 is not altered, since self punishment by the principal occurs out of the equilibrium path).

3.7 Applications

Organization of the Firm

From our discussion in the previous sections, we conclude that auditing is optimal when the punishment instrument is strong for any information structure (except when the hard and forgeable or soft information is noisy). Instead, there is more room for a supervisor when the punishment instrument is weak and his information is very informative about the agent's type (regardless of the quality), or when his information is noisy and soft.

These results are consistent with typical organizational structures, in which low-level workers (typically with lower incomes or protected by minimum wages) are supervised during production stage, while top-level managers (such as CEOs or top-level managers, who typically are able to respond to fines up to some level) are exposed to audits.

As we mentioned before, it is reasonable to think that information from supervisors is "softer", while information from auditors may be "forgeable" (See Section 3.6). Hence, the results in the previous paragraph extend to these more general scenarios.

Regulation

The literature on regulation and information (Laffont and Tirole [43] and others) has studied ex ante and ex post regulation separately (which provides the building blocks for this work). The theoretical framework developed here nests both regulatory stages. Table 3.1 in the Introduction provides a summary of the optimal regulatory stage under different information environments when collusion is present (see page 75).

The Law and Economics literature (see Shavell [53], Kolstad *et al.* [39]) has studied the optimal regulatory stage of activities that generate externalities with a benevolent regulator. Consider, for example, the case of a hazardous activity with "disastrous" consequences. From Section 3.3.2 (in which we add incentives), and in accordance with the standard recommendation, the government should put all the efforts in ex ante regulation whenever the liability faced by the injurer is low (as it is the case when the bad outcome involves irremediable consequences).[29] Ex post regulation is recommended when the injurer can be strongly punished. This recommendation may be modified when the regulatory agency is self-interested. When the regulator's

[29] For example, Cohen [9], pp. 45-46, shows estimates of very low penalties compared to the environmental damage done by oil spills.

information concerning the damages caused by the injurer can be manipulated, there are circumstances under which the ex post regulation is no longer optimal even when punishment is strong. The high stakes in collusion and a noisy report are reasons for the government to concentrate efforts in preventing unwanted damages through ex ante regulation (see Sections 3.5 and 3.6). In general, we provide another source of enforcement cost: the threat of collusion between the regulatory agency and the private party under different information environments.

3.8 Conclusion

In this chapter we study the case of one-time monitoring in hierarchies. We provide insights on the optimality to the principal of using monitoring timing as a choice variable when collusion is present. Previous literature has analyzed both monitoring cases separately or studied the timing with a benevolent regulator. We show that the monitoring timing decision imposes a trade-off to the principal and provide the solution to this trade-off. Under hard information, a supervisor is optimal when his signal reflects the agent's productivity accurately, and an auditor is optimal when his signal is noisy, provided that the punishment instrument is limited. An auditor is optimal when the principal can expose the agent to severe fines. When information is soft (or hard and forgeable), there are circumstances where a supervisor is optimal when his signal is noisy even for unbounded penalties.

Once we understand the forces that drive the principal to choose auditing or supervising under different collusion environments, it is natural to ask how these effects work when there is repeated interaction among the parties. An important issue that arises from this situation is the availability to the principal of instruments to reduce the noise of the monitor's signal or the manipulation of (hard and forgeable or soft) information.

We assume that the principal has to decide between ex ante or ex post monitoring. However, when the principal has access to both monitors who are not related and do not share information, there is no reason for the principal not to choose both monitors if gathering information is costless.[30] However, when the monitors share some information, additional collusion constraints may arise, which may lead the principal to redesign the optimal contract. This problem becomes relevant when information is forgeable or soft. We leave this case to future research.

There is a more general question of strategic timing. In our framework, the principal optimally chooses to delay the monitoring to later stages when she hires an auditor (in particular, when effort monitoring is not available). This result is a case of strategic timing, since under some circumstances the principal delays the gathering of relevant information that is available in the beginning of the game. Along these lines, there is a broader question that has to do with strategic contracting. Theoretical models assume that grand contracts are designed at the beginning of a general

[30] There is research along these lines (with a benevolent regulatory agency). For example, Kolstad *et al.* [39] show that ex ante and ex post regulation may be complements depending on the injurer's uncertainty of his potential liability.

game. By constraining the set of decisions at some period of the game, the parties may get some benefit at later stages. For example, in a paper on collusion and delegation, Laffont and Martimort [41] show that a principal finds profitable to delegate to the supervisor the direct contracting with an agent.

3.9 Appendix to Chapter 3

Proof of Proposition 10: When the principal hires a benevolent auditor, she pays him $w^r = 0$ for $r \in \{0, L, H\}$. Constraint IC(L) is non-binding when the others are satisfied. The Lagrangian to problem (3.7) is

$$\begin{aligned}
\mathcal{L} &= V - \bar{\theta} + q\{\theta_H e_h - t_h\} + (1-q)\left\{\theta_L e_l - t_l + \left[(1-p)\,z^0 + p\,(1-\alpha)\,z^H\right]\right\} \\
&+ \lambda_1 \left\{t_l - \left[(1-p)z^0 + p(1-\alpha)z^H\right] - \frac{e_l^2}{2}\right\} + \lambda_2 \left\{t_h - \frac{e_h^2}{2}\right\} \\
&+ \lambda_3 \left\{t_h - \frac{e_h^2}{2} - t_l + \left[(1-p)z^0 + p\alpha z^H\right] + \frac{e_l^2 \Delta\theta}{2}\right\}
\end{aligned}$$

The Kuhn-Tucker conditions are

$$\begin{aligned}
&\mathcal{L}_{e_h} = q\theta_H - (\lambda_2 + \lambda_3)\, e_h \leq 0, && e_h \geq 0, && \mathcal{L}_{e_h} e_h = 0 \\
&\mathcal{L}_{e_l} = (1-q)\theta_L - (\lambda_1 - \lambda_3 \Delta\theta)\, e_l \leq 0, && e_l \geq 0, && \mathcal{L}_{e_l} e_l = 0 \\
&\mathcal{L}_{t_h} = -q + \lambda_2 + \lambda_3 \leq 0, && t_h \geq 0, && \mathcal{L}_{t_h} t_h = 0 \\
&\mathcal{L}_{t_l} = -(1-q) + \lambda_1 - \lambda_3 \leq 0, && t_l \geq 0, && \mathcal{L}_{t_l} t_l = 0
\end{aligned}$$

$$\begin{aligned}
\mathcal{L}_{z^0} &= (1-q) - \lambda_1 + \lambda_3 = 0; \text{ if } \mathcal{L}_{z^0} < 0,\ z^0 = 0; \text{ if } \mathcal{L}_{z^0} > 0,\ z^0 = z \\
\mathcal{L}_{z^H} &= (1-q)(1-\alpha) - \lambda_1(1-\alpha) + \lambda_3\alpha = 0 \\
&\quad \text{if } \mathcal{L}_{z^H} < 0,\ z^H = 0; \text{ if } \mathcal{L}_{z^H} > 0,\ z^H = z
\end{aligned}$$

together with the participation and incentive compatibility constraints. The solution involves positive e_l and e_h. From the participation constraints, t_l and t_h are both positive. Then $\mathcal{L}_{e_h} = \mathcal{L}_{t_h} = 0$, which implies that the type-θ_H agent exerts first-best effort $e_h = \theta_H$. Also, $\mathcal{L}_{e_l} = \mathcal{L}_{t_l} = 0$ and

$$(1-q)\theta_L = [\lambda_1 - \lambda_3\Delta\theta]\, e_l \tag{3.25}$$

$$(1-q) = \lambda_1 - \lambda_3 \tag{3.26}$$

Using (3.26), $\mathcal{L}_{z^0} = 0$ and hence $z^0 = z$ without loss of generality. Also, $\mathcal{L}_{z^H} = \lambda_3(2\alpha - 1) \geq 0$ and then $z^H = z$. This is a maximum deterrence result (see Baron and Besanko [2]).

Next we consider the three possible cases for the relationship between IC(H) and IR(H): either of them or both of them are binding.

Case 1: $\lambda_2 = 0$ and $\lambda_3 = q$. IR(H) is non-binding and IC(H) is binding. Using equations (3.25) and (3.26), $e_l = (1-q)\theta_L / [(1-q) + qR]$. Using the IR and IC constraints, $t_l = e_l^2/2 + (1-p\alpha)z$, $t_h = \theta_H^2/2 + e_l^2 R/2 - p(2\alpha-1)z$. This is the solution if IR(H) is non-binding (i.e., $e_l^2 R/2 > p(2\alpha-1)z$), which is satisfied for $\alpha < \alpha_1^*$, where

$$\alpha_1^* = \frac{1}{2} + \frac{(1-q)^2\theta_L^2 R}{4pz\left[(1-q)+qR\right]^2} \tag{3.27}$$

Case 2: $\lambda_2 = q$ and $\lambda_3 = 0$. IR(H) is binding and IC(H) is non-binding. Using (3.26) in (3.25) we have $e_l = \theta_L$. From the IC and IR constraints, $t_l = \theta_L^2/2 + (1-p\alpha)z$, $t_h = \theta_H^2/2$, and IC(H) must hold as inequality (i.e., $\theta_L^2 R/2 < p(2\alpha-1)z$), which is satisfied for $\alpha > \alpha_2^*$, where

$$\alpha_2^* = \frac{1}{2} + \frac{\theta_L^2 R}{2pz} \tag{3.28}$$

Case 3: Both λ_2 and λ_3 are non-negative (and both less than or equal to q). Using IR and IC constraints, the type-θ_H agent's rent must be zero, i.e., $e_l^2 R/2 = p(2\alpha-1)z$. Hence, $e_l = \sqrt{2p(2\alpha-1)z/R}$. Using equations (3.25) and (3.26), $\lambda_3 = (1-q)\left(\theta_L/e_l - 1\right)/R$. The agent compensation is $t_l = \theta_L^2/2 + (1-p\alpha)z$ and $t_h = \theta_H^2/2$.

The Lagrangian is concave since it is linear in compensations and punishments, there are no cross terms among them and efforts, and the second derivative with respect to effort is negative. The summary of effort and compensations is presented in equations (3.8)-(3.9). *Q.E.D.*

Proof of Theorem 1: Define $U_H = V - \bar{\theta} + q\frac{\theta_H^2}{2}$. The principal's utility with a supervisor is

$$EU_P^{NC}(S) = U_H + \frac{(1-q)^2\theta_L^2}{2}\left\{\frac{p\alpha^2}{(1-q)\alpha + q(1-\alpha)R} + \frac{p(1-\alpha)^2}{(1-q)(1-\alpha) + q\alpha R} + \frac{1-p}{(1-q)+qR}\right\} \tag{3.29}$$

$EU_P^{NC}(S)$ is increasing and convex in α. The principal's utility with an auditor is

$$EU_P^{NC}(A) = \begin{cases} U_H + \dfrac{(1-q)^2\theta_L^2}{2\left[(1-q)+qR\right]} + qp(2\alpha-1)z = EU_P^{NM} + qp(2\alpha-1)z & \text{if } \alpha < \alpha_1^* \\ U_H + (1-q)\left\{\sqrt{\dfrac{2p(2\alpha-1)z}{R}}\theta_L - \dfrac{p(2\alpha-1)z}{R}\right\} & \text{if } \alpha_1^* \le \alpha \le \alpha_2^* \\ EU_P^{FB} & \text{if } \alpha > \alpha_2^* \end{cases} \tag{3.30}$$

where α_1^* is from (3.27) and α_2^* is from (3.28), EU_P^{NM} is equation (3.1), EU_P^{FB} is the first-best utility, and the superscript NC stands for no collusion. The first part of this utility function is increasing and linear in α, the second part is concave and the last part is constant. By construction, the principal's utility is continuous in all parameters.

If $z = 0$, $EU_P^{NC}(A) = EU_P^{NM}$, while $EU_P^{NC}(S) > EU_P^{NM}$. On the other hand, when z is very high, $EU_P^{NC}(A) > EU_P^{NC}(S)$ for all α. So, for intermediate values of z, $EU_P^{NC}(S)$ intersects $EU_P^{NC}(A)$ at some cut-off value α^C such that $EU_P^{NC}(S) > EU_P^{NC}(A)$ for $\alpha < \alpha^C$, and $EU_P^{NC}(A) < EU_P^{NC}(S)$ for $\alpha > \alpha^C$. Also, as z increases $EU_P^{NC}(A)$ increases, while $EU_P^{NC}(S)$ remains the same for a given α, and hence there exists a critical liability bound $\bar{z}$ such that $EU_P^{NC}(A) > EU_P^{NC}(S)$ for all α when $z > \bar{z}$. *Q.E.D.*

Proof of Result 1: The region of optimality of a supervisor expands out as adverse selection is more severe (i.e., R increases, caused by an increase in θ_H). The proof is done assuming that parameters are such that $\alpha_1^* \geq 1$ (from equation (3.27)) and $q \leq 1/(1+R)$. We show that $EU_P^{NC}(S)$ decreases less than $EU_P^{NC}(A)$ does as R increases. When this is the case, the new intersection occurs at a lower value of α (keeping all the other parameters fixed). From equations (3.29) and (3.30), eliminate the common parts U_H and $(1-q)^2\theta_L^2/2$ to get that

$$\frac{\partial}{\partial R}\left(\frac{p\alpha^2}{(1-q)\alpha + q(1-\alpha)R} + \frac{p(1-\alpha)^2}{(1-q)(1-\alpha)+q\alpha R} + \frac{1-p}{(1-q)+qR}\right) > \frac{\partial}{\partial R}\left(\frac{1}{[(1-q)+qR]}\right) \tag{3.31}$$

which, after several steps (omitted for convenience), simplifies to

$$\alpha(1-\alpha)\left\{\left[2\left(\alpha^2+(1-\alpha)^2\right)-4\alpha\,(1-\alpha)\right](1-q)^3+\left[\alpha\,(1-\alpha)-\alpha^3-(1-\alpha)^3\right]q^3R^3\right\}$$
$$+\left[\alpha^4+(1-\alpha)^4+2\alpha^2(1-\alpha)^2-\alpha\,(1-\alpha)\right](1-q)^2qR > 0$$

The first term is positive, the second term is non-negative and the third term is non-positive. But note that $(1-q)^2q^2 \geq q^3R^3$ and $(1-q)^3 \geq q^3R^3$ when $q \leq 1/(1+R)$, and that the sum of all brackets simplifies to $1-4\alpha+4\alpha^2 > 0$, for $\alpha > 1/2$. Hence, inequality (3.31) is satisfied. *Q.E.D.*

Constraints for an optimal contract under collusion with hard and non-forgeable information: We first simplify the conditions to be satisfied by an optimal contract, and then show the *simplified* coalition constraints (3.11). In all cases, $j \in \{L, H\}$ and $r \in \{0, L, H\}$. A feasible contract must satisfy the agent's participation and incentive constraints (3.2), repeated here for convenience.

$$\begin{aligned} &\text{IR(jr)}: \quad t_{jr} \geq e_{jr}^2/2 \\ &\text{IC(Hr)}: \quad t_{Hr} - e_{Hr}^2/2 \geq t_{Lr} - e_{Lr}^2 \Delta\theta/2 \end{aligned} \tag{3.32}$$

Also, it must satisfy the supervisor's limited liability constraints ($w_{jr} \geq 0$) and the agent-supervisor coalition constraints. These constraints are as follows. The supervisor's degree of discretion is to conceal his signal. At the time of meeting the agent, the supervisor does not know the agent's type, and hence has updated beliefs

$$Pr(\theta_H|\sigma = H) = \frac{q\alpha}{q\alpha + (1-q)(1-\alpha)} \qquad Pr(\theta_L|\sigma = H) = \frac{(1-q)(1-\alpha)}{q\alpha + (1-q)(1-\alpha)}$$
$$Pr(\theta_H|\sigma = L) = \frac{q(1-\alpha)}{q(1-\alpha) + (1-q)\alpha} \qquad Pr(\theta_L|\sigma = L) = \frac{(1-q)\alpha}{q(1-\alpha) + (1-q)\alpha}$$

In order to avoid concealment of information, the principal has to pay the coalition a higher wage when the monitor's signal is either L or H than that when the signal is 0, i.e.,

$$\begin{aligned} (H) \quad & Pr(\theta_H|\sigma = H)\,[w_{HH} + kR_{HH}] + Pr(\theta_L|\sigma = H)\,[w_{LH} + kR_{LH}] \geq \\ & Pr(\theta_H|\sigma = H)\,[w_{H0} + kR_{H0}] + Pr(\theta_L|\sigma = H)\,[w_{L0} + kR_{L0}] \\ (L) \quad & Pr(\theta_H|\sigma = L)\,[w_{HL} + kR_{HL}] + Pr(\theta_L|\sigma = L)\,[w_{LL} + kR_{LL}] \geq \\ & Pr(\theta_H|\sigma = L)\,[w_{H0} + kR_{H0}] + Pr(\theta_L|\sigma = L)\,[w_{L0} + kR_{L0}] \end{aligned} \tag{3.33}$$

In an optimal contract, the principal leaves no rents to the type-θ_L agent ($R_{Lr} = 0$, since the latter does not have incentives to individually misreport her type), and pays $t_{Lr} = e_{Lr}^2/2$. Constraints IC(Hr) are also binding (standard result), and $t_{Hr} =$

$e_{Hr}^2/2 + R_{Hr}$, where $R_{Hr} = Re_{Lr}^2/2$. Since the agent earns no rent in states L0, LL and LH, the principal can pay $w_{L0} = w_{LL} = w_{LH} = 0$ to the supervisor (the coalition does not benefit from changing the supervisor's report given the agent's type). Using all these results in (3.33), we have that

$$\text{CC(HH)}: \qquad w_{HH} + kR_{HH} \geq w_{H0} + kR_{H0}$$

$$\text{CC(HL)}: \qquad w_{HL} + kR_{HL} \geq w_{H0} + kR_{H0}$$

These are equations (3.11) in the text. Hence, a contract that solves the principal problem subject to the constraints IR(Lr) and IC(Hr) (from (3.2)), limited liability (3.10), and (3.11) also solves the problem with the more general constraints (3.33).

Q.E.D.

Proof of Proposition 11:[31] The Proof of Condition (3.11) contains the relevant constraints to be satisfied in an optimal contract. In all cases, $r \in \{0, L, H\}$. The participation constraints IR(Lr) and incentive constraints IC(Hr) are binding (and then it is straightforward to show that constraints IR(Hr) and IC(Lr) hold strictly). Then $R_{Lr} = 0$ and $R_{Hr} = Re_{Lr}^2/2$. The supervisor's compensation is such that $w_{Lr} = 0$ and $w_{H0} = 0$ (in this last case, the monitor has no information and cannot forge it). We solve the optimization problem making constraint CC(HH) binding, and then check that constraint CC(HL) is always non-binding with $w_{HL} = 0$.

Consider first that $p < 1$. The principal's problem (3.12) simplifies to maximize

$$V - \bar{\theta} + \sum_{r \in \{0,L,H\}} \left\{ \pi_{Hr} \left[\theta_H e_{Hr} - \frac{e_{Hr}^2}{2} - R\frac{e_{Lr}^2}{2} \right] + \pi_{Lr} \left[\theta_L e_{Lr} - \frac{e_{Lr}^2}{2} \right] \right\} - \pi_{HH} Rk \left(\frac{e_{L0}^2}{2} - \frac{e_{LH}^2}{2} \right)$$

This program is concave. From the first-order conditions for a maximum we have

$$e_{H0} = e_{HL} = e_{HH} = \theta_H \qquad e_{L0} = \frac{(1-q)(1-p)\theta_L}{(1-q)(1-p) + qR\left[(1 - p(1-\alpha k)\right]}$$

$$e_{LL} = \frac{(1-q)\alpha\theta_L}{(1-q)\alpha + qR(1-\alpha)} \qquad e_{LH} = \frac{(1-q)(1-\alpha)\theta_L}{(1-q)(1-\alpha) + qR\alpha(1-k)}$$

[31] In the proof of Propositions 11 and 13 we will simplify the state probabilities as $\pi_{L0} = (1-q)(1-p)$, $\pi_{LL} = (1-q)p\alpha$, $\pi_{LH} = (1-q)p(1-\alpha)$, $\pi_{H0} = q(1-p)$, $\pi_{HL} = qp(1-\alpha)$, and $\pi_{HH} = qp\alpha$.

It is easy to check that $e_{LL} > e_{L0}$ and hence $R_{HL} > R_{H0}$. Constraint CC(HL) holds strictly with $w_{HL} = 0$. On the other hand, $e_{L0} > e_{LH}$ (and then $R_{H0} > R_{HH}$) if and only if $\alpha > \alpha_h$, where

$$\alpha_h = \begin{cases} \dfrac{-[2(1-p)-k] + \sqrt{[2(1-p)-k]^2 + 4pk(1-p)}}{2pk} & \text{if} \quad k > 0 \\ 1/2 & \text{if} \quad k = 0 \end{cases} \tag{3.34}$$

When $\alpha \leq \alpha_h$, effort and compensations are such that $e_{LH} = e_{L0}$, $t_{HH} = t_{H0}$ and $w_{HH} = 0$ (that is, CC(HH) is binding at $w_{HH} = 0$), where

$$e_{L0} = e_{LH} = \frac{(1-q)(1-p\alpha)\theta_L}{(1-q)(1-p\alpha) + qR(1-p(1-\alpha))}$$

When $p = 1$, concealment is not possible and hence the coalition constraints are no longer relevant. The solution to the principal's problem is as in (3.4)-(3.5).

From the results above, the principal always hires the supervisor. Effort and compensations are summarized in equations (3.14)-(3.15). *Q.E.D.*

Proof of Proposition 13: The proof of the Proposition is similar to that of Proposition 11. In all cases, $j \in \{L, H\}$ and $r \in \{0, L, H\}$. First, we solve the optimal contract with the binding constraints IR(Lr), IC(Hr), CC(H0-HL) and CC(HH-HL) (from equations (3.2) and (3.17)), and then we check the conditions under which the other constraints are strictly satisfied. Without loss of generality we can set $w_{Lr} = 0$. Also, $w_{HL} = 0$ since this is the most expensive state to pay the monitor a collusion-proof wage. We summarize the state probabilities as π_{jr} (see footnote 31 in page 122). The principal's problem is to maximize ($V - \bar{\theta}$ is subtracted)

$$\sum_{r \in \{0,L,H\}} \pi_{Lr} \left\{ \theta_L e_{Lr} - \frac{e_{Lr}^2}{2} \right\} + \sum_{r \in \{0,L,H\}} \pi_{Hr} \left\{ \theta_H e_{Hr} - \frac{e_{Hr}^2}{2} - R\frac{e_{Lr}^2}{2} - Rk\left(\frac{e_{LL}^2}{2} - \frac{e_{Lr}^2}{2} \right) \right\}$$

The program is concave. From the first-order conditions for a maximum we have:

$$e_{H0} = e_{HL} = e_{HH} = \theta_H \qquad e_{LL} = \frac{(1-q)p\alpha\theta_L}{(1-q)p\alpha + qR[k + p(1-\alpha)(1-k)]}$$

$$e_{L0} = \frac{(1-q)\theta_L}{(1-q) + qR(1-k)} \quad e_{LH} = \frac{(1-q)(1-\alpha)\theta_L}{(1-q)(1-\alpha) + q\alpha R(1-k)}$$

Note that $e_{L0} > e_{LH}$. Now, $e_{LL} > e_{L0}$ if and only if $\alpha > \alpha_2^f$, where

$$\alpha_2^f = \frac{k + (1-k)p}{2p(1-k)} \tag{3.35}$$

The second case is $e_{LL} = e_{L0} > e_{LH}$ (for $\alpha \leq \alpha_2^f$). Plugging this restriction into the program and solving for effort levels, we have e_{LH} as before and

$$e_{L0} = e_{LL} = \frac{(1-q)(1-p(1-\alpha))\theta_L}{(1-q)(1-p(1-\alpha)) + qR\left[1 - p\alpha(1-k)\right]}$$

Next, $e_{L0} = e_{LL} > e_{LH}$ if $\alpha_1^f < \alpha \leq \alpha_2^f$, where

$$\alpha_1^f = \frac{1}{2-k} \tag{3.36}$$

Otherwise, the solution involves not hiring the supervisor for $\alpha \leq \alpha_1^f$. The results are summarized in equations (3.19)-(3.20). *Q.E.D.*

Proof of Proposition 14: An optimal contract must satisfy (3.6), (3.21) and the limited liability constraints. Constraints (3.21) are binding (otherwise, there are benefits from joint deviations). Also, $w^L = 0$ (since eventual forgery is to change the monitor's report to $r = L$) and $w^r = kz^r$ (the principal obtains an honest report), for $r \in \{0, H\}$. The principal sends the (costly) auditor with probability $\delta \in (0, 1]$. The Lagrangian of the principal's problem is ($V - \bar{\theta}$ is omitted):

$$\begin{aligned}\mathcal{L} \;=\; & q\left\{\theta_H e_h - t_h\right\} + (1-q)\left\{\theta_L e_l - t_l + \delta(1-k)\left[(1-p)z^0 + p(1-\alpha)z^H\right]\right\} \\ & + \lambda_1\left\{t_l - \delta\left[(1-p)z^0 + p(1-\alpha)z^H\right] - \frac{e_l^2}{2}\right\} + \lambda_2\left\{t_h - \frac{e_h^2}{2}\right\} \\ & + \lambda_3\left\{t_h - \frac{e_h^2}{2} - t_l + \delta\left[(1-p)z^0 + p\alpha z^H\right] + \frac{e_l^2\Delta\theta}{2}\right\} + \lambda_4\{1-\delta\}\end{aligned}$$

The Kuhn-Tucker conditions are

$$\mathcal{L}_{e_h} = q\theta_H - (\lambda_2 + \lambda_3)\, e_h \leq 0, \qquad e_h \geq 0, \qquad \mathcal{L}_{e_h} e_h = 0$$

$$\mathcal{L}_{e_l} = (1-q)\theta_L - (\lambda_1 - \lambda_3 \Delta\theta)\, e_l \leq 0, \qquad e_l \geq 0, \qquad \mathcal{L}_{e_l} e_l = 0$$

$$\mathcal{L}_{t_h} = -q + \lambda_2 + \lambda_3 \leq 0, \qquad t_h \geq 0, \qquad \mathcal{L}_{t_h} t_h = 0$$

$$\mathcal{L}_{t_l} = -(1-q) + \lambda_1 - \lambda_3 \leq 0, \qquad t_l \geq 0, \qquad \mathcal{L}_{t_l} t_l = 0$$

$$\mathcal{L}_{z^0} = (1-q)(1-k) - \lambda_1 + \lambda_3 = 0, \quad \text{if } \mathcal{L}_{z^0} < 0, z^0 = 0, \quad \text{if } \mathcal{L}_{z^0} > 0, \quad z^0 = z$$

$$\mathcal{L}_{z^H} = (1-q)(1-\alpha)(1-k) - \lambda_1(1-\alpha) + \lambda_3 \alpha = 0,$$

$$\text{if} \quad \mathcal{L}_{z^H} < 0, \quad z^H = 0, \quad \text{if} \quad \mathcal{L}_{z^H} > 0, z^H = z$$

$$\mathcal{L}_\delta \;=\; \left[(1-p)z^0 + p(1-\alpha)z^H\right]\left[(1-q)(1-k) - \lambda_1\right] + \lambda_3\left[(1-p)z^0 + p\alpha z^H\right] - \lambda_4 = 0,$$

$$\text{if} \quad \mathcal{L}_\delta < 0, \delta = 0, \qquad \text{if} \quad \mathcal{L}_\delta > 0, \delta = 1$$

together with the participation, incentive and coalition constraints. The solution involves positive e_l and e_h. From the participation constraints, t_l and t_h are both positive. Then $\mathcal{L}_{e_h} = \mathcal{L}_{e_l} = \mathcal{L}_{t_h} = \mathcal{L}_{t_l} = 0$ and $e_h = \theta_H$. Also

$$(1-q)\theta_L \;=\; \left[\lambda_1 - \lambda_3 \Delta\theta\right] e_l \tag{3.37}$$

$$(1-q) \;=\; \lambda_1 - \lambda_3 \tag{3.38}$$

Using (3.38), $\mathcal{L}_{z^0} = -(1-q)k < 0$ and then $z^0 = 0$. On the other hand,

$$\mathcal{L}_{z^H} = \lambda_3(2\alpha - 1) - (1-q)(1-\alpha)k \tag{3.39}$$

The relevant cases from a combination of λ_2 and λ_3 (such that $\lambda_2 + \lambda_3 = q$) are:

* <u>Case 1:</u> The highest value for λ_3 is q. An auditor is not profitable, and the principal contracts with the agent directly (No-Monitor-Contract), when (3.39) is negative. This is true for $\alpha < \alpha_1$, where

$$\alpha_1 = \frac{q + (1-q)k}{2q + (1-q)k} \tag{3.40}$$

Otherwise, $z^H = z$ and the principal hires the auditor in the remaining cases.

* Case 2: $\lambda_2 = 0, \lambda_3 = q, \delta = 1$. Using (3.37), IR, IC and CC constraints,

$$\begin{aligned} e_l &= \frac{(1-q)\theta_L}{(1-q)+qR}, \qquad t_l = e_l^2/2 + p(1-\alpha)z \\ t_h &= \theta_H^2/2 + Re_l^2/2 - p(2\alpha-1)z, \qquad w^0 = w^L = 0, \quad w^H = kz \end{aligned}$$

Plugging this information in $\mathcal{L}_\delta$, we have that $\delta = 1$. The type-θ_H agent's rent is $R_h = e_l^2/2 - p(2\alpha-1)z$. This is the solution if the agent earns a positive rent ($R_h > 0$), which, after some algebra, simplifies to $\alpha_1 \leq \alpha < \alpha_2$,[32] where α_2 is

$$\alpha_2 = \frac{1}{2} + \frac{(1-q)^2\theta_L^2 R}{4pz\left[(1-q)+qR\right]^2} \tag{3.41}$$

* Case 3: $\lambda_2 > 0, \lambda_3 > 0, \delta = 1$. Using (3.37), IR, IC and CC constraints (with $R_h = 0$),

$$\begin{aligned} e_l &= \sqrt{2pz(2\alpha-1)/R}, \qquad t_l = e_l^2/2 + p(1-\alpha)z, \qquad t_h = \theta_H^2/2 \\ w^0 &= w^L = 0, \quad w^H = kz, \qquad \lambda_3 = (1-q)\left(\theta_L/e_l - 1\right)/R \end{aligned}$$

The probability δ is equal to 1 if $\mathcal{L}_\delta \geq 0$, which after tedious algebra reduces to $\alpha_2 \leq \alpha \leq \alpha_3$, where α_3 solves the following quadratic equation

$$2pz\left[(2\alpha-1)+(1-\alpha)Rk\right]^2 = (2\alpha-1)R\theta_L^2 \tag{3.42}$$

* Case 4: When $\alpha > \alpha_3$, equation (3.42) does not hold, and the principal sets $\delta < 1$ (where $\mathcal{L}_\delta = 0$). Higher punishment implies higher collusion cost, which can be minimized by reducing the auditing probability. The solution for this case is

$$\begin{aligned} e_l &= \frac{(2\alpha-1)\theta_L}{(2\alpha-1)+(1-\alpha)kR}, \quad \lambda_3 = \frac{(1-q)(1-\alpha)k}{2\alpha-1}, \quad t_l = \frac{e_l^2}{2} + \delta p(1-\alpha)z \\ t_h &= \frac{\theta_H^2}{2}, \quad w^0 = w^L = 0, \quad w^H = kz, \quad \delta = \frac{(2\alpha-1)R\theta_L^2}{2pz\left[(2\alpha-1)+(1-\alpha)kR\right]^2} \end{aligned}$$

When $\alpha = 1$ this case reduces to Case 5, in which $\lambda_3 = 0$. Constraint IC(H) holds strictly, i.e., $R_h < 0$, which is satisfied if $z > R\theta_L^2/2p$. The solution is

$$e_l = \theta_L, \qquad t_l = e_l^2/2, \qquad t_h = \theta_H^2/2, \qquad w^0 = w^L = 0, \quad w^H = kz, \qquad \delta \in (0,1]$$

[32] This case does not exist when $\alpha_2 < \alpha_1$.

The contract is summarized in equations (3.23)-(3.24). *Q.E.D.*

Proof of Theorem 2: Assume parameters such that $\alpha_1^f < \alpha_1$ (from (3.27) and (3.40), respectively), that is, $(1-2q) > k(1-q)$. The principal's utility with a supervisor (from contract (3.19)-(3.20)) is

$$EU_P^{FI}(S) = \begin{cases} EU_P^{NM} & \text{if } \alpha < \alpha_1^f \\ U_H + \frac{(1-q)^2 p(1-\alpha)^2 \theta_L^2}{2[(1-q)(1-\alpha)+q\alpha R(1-k)]} + \frac{(1-q)^2(1-p(1-\alpha))^2\theta_L^2}{2[(1-q)(1-p(1-\alpha))+qR(1-p\alpha(1-k))]} & \text{if } \alpha_1^f \leq \alpha < \alpha_2^f \\ U_H + \frac{(1-q)^2(1-p)\theta_L^2}{2[(1-q)+qR(1-k)]} + \frac{(1-q)^2 p^2\alpha^2\theta_L^2}{2[(1-q)p\alpha+qR(k+p(1-\alpha)(1-k))]} + \frac{(1-q)^2 p(1-\alpha)^2\theta_L^2}{2[(1-q)(1-\alpha)+q\alpha R(1-k)]} & \text{if } \alpha_2^f \leq \alpha < 1 \\ U_H + \frac{(1-q)^2(1-p)\theta_L^2}{2[(1-q)+qR(1-k)]} + \frac{(1-q)^2 p^2\theta_L^2}{2[(1-q)p+qRk]} & \text{if } \alpha = 1 \end{cases} \tag{3.43}$$

where $U_H = V - \bar{\theta} + q\theta_H^2/2$, EU_P^{NM} is equation (3.1), EU_P^{FB} is the first-best utility, and the superscript FI stands for forgeable information. After tedious algebra it can be shown that $EU_P^{FI}(S)$ is continuous in α. For $\alpha > \alpha_1^f$, it is increasing and strictly convex in α, $EU_P^{FI}(S) > EU_P^{NM}$, and $EU_P^{FI}(S) < EU_P^{FB}$ (for $p < 1$ or $k > 0$). $EU_P^{FI}(S)$ does not depend on z.

Similarly, the principal's utility with an auditor (from contract (3.23)-(3.24)) is

$$EU_P^{FI}(A) = \begin{cases} EU_P^{NM} & \text{if } \alpha < \alpha_1 \\ U_H + \frac{(1-q)^2\theta_L^2}{2[(1-q)+qR]} + pz\left\{q(2\alpha-1) - (1-q)(1-\alpha)k\right\} & \text{if } \alpha_1 \leq \alpha < \alpha_2 \\ U_H + (1-q)\left\{\sqrt{\frac{2p(2\alpha-1)z}{R}}\theta_L - \frac{p(2\alpha-1)z}{R} - p(1-\alpha)zk\right\} & \text{if } \max\{\alpha_1, \alpha_2\} \leq \alpha \leq \alpha_3 \\ U_H + (1-q)\frac{(2\alpha-1)\theta_L^2}{2[(2\alpha-1)+(1-\alpha)Rk]} & \text{if } \alpha_3 < \alpha \\ EU_P^{FB} & \text{if } \alpha = 1 \text{ and } z > R\theta_L^2/2p \end{cases} \tag{3.44}$$

$EU_P^{FI}(A)$ is continuous in α. For $\alpha > \alpha_1$, it is increasing and concave in α (in particular, it is linear for $\alpha \in [\alpha_1, \alpha_2)$), $EU_P^{FI}(A) > EU_P^{NM}$. Finally, $EU_P^{FI}(A) = EU_P^{FB}$ for $\alpha = 1$ and $z > R\theta_L^2/2p$.

Fix all parameters and vary z to show the following results:

a) When $z = 0$, $EU_P^{FI}(S) > EU_P^{FI}(A)$ $(= EU_P^{NM})$, for $\alpha > \alpha_1^f$. Supervising is optimal in this range of α. The same result holds as z increases (until z reaches a value $\bar{z}_1$).

b) When $z > \bar{z}_1$, there are two cut-off values of α, α_1^C and α_2^C, such that $EU_P^{FI}(S) > EU_P^{FI}(A)$ for $\alpha_1^f < \alpha < \alpha_1^C$ and for $\alpha > \alpha_2^C$, while $EU_P^{FI}(A) > EU_P^{FI}(S)$ for $\alpha_1^C < \alpha < \alpha_2^C$. As z increases from $\bar{z}_1$, $EU_P^{FI}(A)$ increases for all $\alpha > \alpha_1$, and

EU_P^S remains the same. Consequently, α_1^C decreases and α_2^C increases. Define $\bar{z}_2$ the value of the liability bound such that $\alpha_2^C = 1$.

c) As z increases from $\bar{z}_2$, α_2^C remains equal to 1 (since at $\alpha = 1$, $EU_P^{FI}(A) > EU_P^{FI}(S)$) and α_1^C decreases up to α_1 (neither α_1^f nor α_1 depends on z).

Finally, when parameters are such that $\alpha_1^f \geq \alpha_1$ (i.e.,$(1-2q) \leq k(1-q)$), α_1^C does not exist. Results a)-c) hold for α_2^C. *Q.E.D.*

Proof of Result 2:

The region of optimality of a supervisor is non-decreasing in the degree of adverse selection R (which increases when θ_H increases). The proof is done assuming parameters such that $\alpha_1^f > \alpha_1$ (that is, $(1-2q) > k(1-q)$), $\alpha_2 \geq 1$ (that is, the agent receives some rent when audited), and $q \leq 1/(1+R)$. We show that the decrease in $EU_P^{FI}(S)$ is less than or equal to that in $EU_P^{FI}(A)$ as R increases. When this happens, the cut-off α_1^C is non-decreasing, and α_2^C is non-increasing, in R (that is, both S_1 and S_2 are non-decreasing in R). Also we have to consider two cases:

i) The contract with a supervisor is such that $\alpha_f^2 \geq 1$. Define $L0 = (1-q)(1-p(1-\alpha)) + qR(1-p\alpha(1-k))$ and $LH = (1-q)(1-\alpha) + q\alpha R\,(1-k)$. After tedious algebra,[33] it is shown that $\partial EU_P^{FI}(S)/\partial R \geq \partial EU_P^{FI}(A)/\partial R$, which implies that

$$\frac{\partial}{\partial R}\left(\frac{(1-p(1-\alpha))^2}{L0} + \frac{p(1-\alpha)^2}{LH}\right) \geq \frac{\partial}{\partial R}\left(\frac{1}{(1-q)+qR}\right)$$

This result holds as weak inequality.

ii) The contract with a supervisor is such that $\alpha_f^2 < 1$. Define $L0 = (1-q) + qR\,(1-k)$, $LL = (1-q)p\alpha + qR\,(k + p(1-\alpha)(1-k))$ and $LH = (1-q)(1-\alpha) + q\alpha R\,(1-k)$. After tedious algebra, it is shown that $\partial EU_P^{FI}(S)/\partial R > \partial EU_P^{FI}(A)/\partial R$, which implies that

$$\frac{\partial}{\partial R}\left(\frac{(1-p)}{L0} + \frac{p\alpha^2}{LL} + \frac{p(1-\alpha)^2}{LH}\right) > \frac{\partial}{\partial R}\left(\frac{1}{(1-q)+qR}\right)$$

This result holds as strict inequality.

The effects of a change in the liability bound z on the monitoring timing can be seen in Proof of Theorem 2 (Figure 3.3).

Now consider a change in the efficiency of side transfers. It is easy to check that $S_2\ (= 1-\alpha_2^C)$ decreases in k, given that $EU_P^{FI}(A)$ at $\alpha = 1$ is not affected by k, while

[33] All the steps to arrive to this result are available upon request.

$EU_P^{FI}(S)$ at $\alpha = 1$ decreases in k. On the other hand, define the interval $\hat{S}_1 = \alpha_1 - \alpha_1^f$ (from (3.36) and (3.40)), which is the subset of S_1 for which an auditor is not hired.

$$\hat{S}_1 = \frac{k\left\{(1-2q)-(1-q)k\right\}}{(2-k)\left(2q+(1-q)k\right)}$$

There are opposing effects of k on $\hat{S}_1$ in both numerator and denominator. The interval $\hat{S}_1$ is more probable to increase (decrease) for low (high) k. *Q.E.D.*

Proof of Theorem 3:

a) Supervisor: The agent's information corresponds to her type and the supervisor's signal. The supervisor's information is his signal.

Consider the following direct mechanism: The principal asks a message report $a = (a_A, a_\sigma) \in \{\theta_L, \theta_H\} \times \{0, L, H\}$ to the agent and a message report $r \in \{0, L, H\}$ to the supervisor, and sets allocations and compensations according the following rule: $\rho(m) = \{e_m, t_m, w_m\}$ (where $m = (a, r)$, and e, t, and w are from equations (3.19)-(3.20)),[34] such that

$$\rho(m,x) = \begin{cases} \{0,0,0\} & \text{if } a_\sigma \neq r \\ \{e_{Lr}, t_{Lr}, 0\} & \text{if } a = (\theta_L, r) \text{ and } r \in \{0, L, R\} \quad (a_\sigma = r) \\ \{e_{Hr}, t_{Hr}, k(t_{HL} - t_{Hr})\} & \text{if } a = (\theta_H, r) \text{ and } r \in \{0, L, R\} \quad (a_\sigma = r) \end{cases}$$

The agent exerts effort e such that $e \in argmax \quad t(m) - e^2/2$. Her strategy is $\zeta_A(\theta,\sigma) = (a(\theta,\sigma), e(\theta,\sigma))$. The supervisor's strategy is $\zeta_M(\sigma) = r(\sigma)$. Then $\zeta = (\zeta_A(\theta,\sigma), \zeta_M(\sigma))$.

Let a strategy profile ζ and set of beliefs ν be a Perfect Bayesian Equilibrium if players do not have incentives to change their strategy at any information set given beliefs, the other players' strategy, and beliefs are updated according to Bayes's rule whenever possible.

Let a side transfer b from the agent to the supervisor be feasible for a given signal σ and manipulation of reports $m' = (a', r')$, if i) $t(m') \geq b \geq -w(m')$ and ii) the agent exerts effort $e' \in argmax \quad t(m') - e'^2/2$.

[34] The value the cost C is recovered from the type report and effort recommendation from that mechanism.

A strategy $s = (m')$ is individually profitable for the supervisor if $w(s) > w(\zeta|\sigma)$, and it is individually profitable for the agent if $t(s) - e'^2/2 > t(m(\theta,\sigma),\theta) - e(\theta,\sigma)^2/2$.

Let a collusive strategy $cs = (m', b)$ be coalition profitable for a signal σ and strategy ζ if a) it is feasible and b) the parties are *strictly better off*, i.e,

i) $t(m') - e'^2/2 - b > t(m(\theta,\sigma),\theta) - e(\theta,\sigma)^2/2$

ii) $w(m') + b > w(\zeta|\sigma)$

Definition: A strategy ζ is Collusion-Proof equilibrium if it is Perfect Bayesian Equilibrium and if there is no feasible and individually or coalition profitable strategy for supervisor and agent under any signal.

According to the allocation rule above, we show that the agent's and supervisor's truthful report of the private information and the agent's acceptance of the principal's effort recommendation is an equilibrium strategy ζ that satisfies collusion proofness. On the one hand, neither the agent nor the supervisor has incentive to individually deviate. If their reports of the supervisor's signal differ ($a_\sigma \neq r$), they will get no utility. The agent does not have incentives to change her type report (the rule satisfies participation and incentive constraints).

On the other hand, we show that the agent-supervisor coalition does not find a feasible deviation. When the agent's type is θ_L, she earns no rent and the monitor's wage is 0. Any mutual change of their report of the monitor's signal is not profitable. A possible deviation may involve the agent changing her report to θ_H and the coalition changing their report to any r. In this case the agent gets a negative rent since $t_{Hr} - e_{Hr}/2\Delta\theta < 0$ for any supervisor's message r. So this deviation could be possible if the supervisor pays the agent up to his wage.[35] The agent's utility in this case is

$$t_{Hr} - \frac{e_{Hr}^2}{2\Delta\theta} + kw_{Hr} = -\frac{R\theta_H^2}{2\Delta\theta} + (1-k^2)\frac{Re_{Lr}^2}{2} + k^2\frac{Re_{LL}^2}{2} < 0$$

for any supervisor's message r. Then any message change a' that involves the agent modifying her report from θ_L to θ_H is not profitable.

Suppose that the type-θ_H agent induces the supervisor to change their report of the supervisor's signal (to some $a'_\sigma = r'$). This is profitable from states H0 or HH to HL and from HH to H0. In all cases the agent gains the difference in her wage, but this is the exact amount needed to compensate the supervisor's wage reduction, which violates conditions i) and/or ii) of coalition profitability.

[35] We also assume that a side transfer b from the monitor (supervisor or auditor) is valued kb by the agent.

b) Auditor: Consider a simplified version of the previous mechanism in which effort is exerted before the auditing stage and only the type-θ_L agent is audited. Now t and w (and z) are from equations (3.23)-(3.24). Any change of their joint report of the monitor's signal from r to $\tilde{r}$ involves a change in agent's utility of $\Delta U_A = (z^{\tilde{r}} - z^r)$ and a change in auditor's utility of $\Delta U_M = -k\Delta U_A = k(z^r - z^{\tilde{r}})$. Suppose that $\Delta U_A > 0$. Then this is the exact amount needed to compensate the auditor's utility loss. On the other hand, suppose that $\Delta U_A < 0$. The maximum side transfer is ΔU_M, valued by the agent at $k\Delta U_M = k^2\Delta U_A$. The agent's utility change is $\Delta U_A - \Delta U_M = (1-k^2)\Delta U_A < 0$. Then, no message deviation is profitable. *Q.E.D.*

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